MA 732 S. Schecter

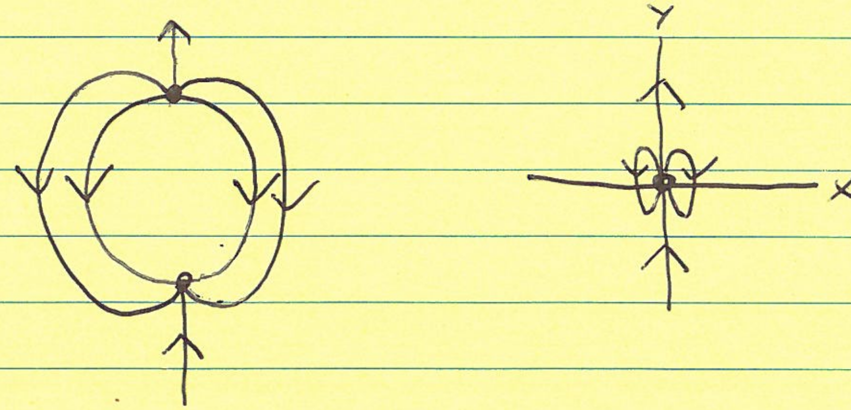
Test 2 Answers

① $\dot{r} = r\sin\theta \qquad \dot{\theta} = -\cos\theta$

a) Equilibria: $(r,\theta) = (0, \frac{\pi}{2}), (0, -\frac{\pi}{2})$

b) $(0, \frac{\pi}{2})$ is repeller, $(0, -\frac{\pi}{2})$ is attractor

c)



② $\dot{x} = f(x,\mu) = x\,a(x,\mu)$

a) All equilibria are $x=0$ or solutions of $a(x,\mu)=0$.

Since $a(0,0)=0$ and $a_x(0,0) \neq 0$, by the Implicit Function Theorem, near $(0,0)$ all sol'ns of $a(x,\mu)=0$ lie on a curve $x = k(\mu)$.

b) $a(k(\mu), \mu) = 0$ and $k(\mu) = b\mu + c\mu^2 + \cdots$

$$\Rightarrow A(b\mu + c\mu^2 + \cdots) + B(b\mu + c\mu^2 + \cdots)^2$$

$$+ C(b\mu + c\mu^2 + \cdots)\mu + D\mu^2 + \cdots = 0$$

μ: $Ab = 0$. Since $A \neq 0$, $b = 0$

μ^2: $Ac + Bb^2 + Cb + D = 0$. Since $b = 0$ and $A \neq 0$,

$$c = -\frac{D}{A}.$$

c) $f_x(x, \mu) = a(x, \mu) + x a_x(x, \mu)$

$f_x(0, \mu) = a(0, \mu) = D\mu^2 + \cdots$

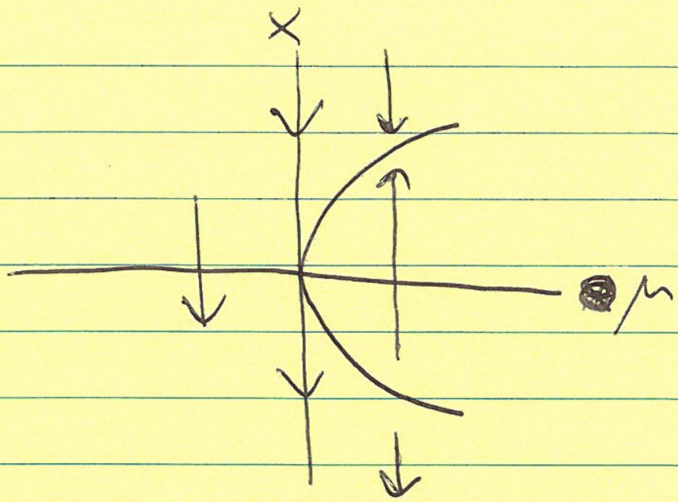
For small $\mu \neq 0$, $f_x(0, \mu)$ has sgn D so $f_x(0, \mu) < 0$.

Therefore for small $\mu \neq 0$, $x = 0$ is an attractor.

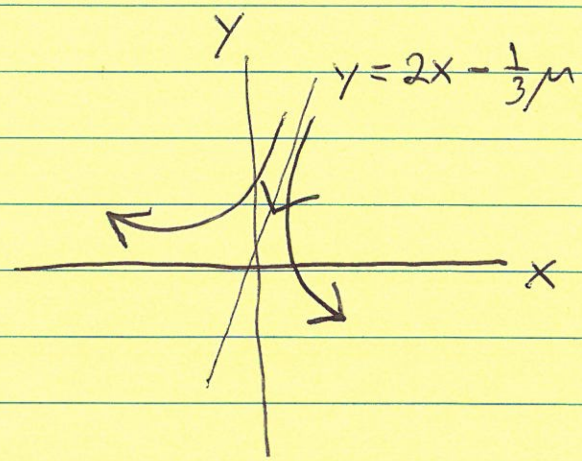
3 a) Center manifold is $y = h(x, \mu) = 2x - \frac{1}{3}\mu - \frac{2}{3}x^2 + \cdots$

b) $\dot{x} = 2x - (2x - \frac{1}{3}\mu - \frac{2}{3}x^2 + \cdots) - x^2 = \frac{1}{3}\mu - \frac{1}{3}x^2 + \cdots$

c)



d) $\mu<0$:



$\mu>0$: