MA 732 Homework 2

S. Schecter

January 23, 2009

1. Let J be an interval, let $B : J \to m \times n$ matrices be continuous and bounded (i.e., there is a number $K \geq 0$ such that $\|B(t\| \leq K$ for all $t \in J$). Define a map $A : C^0(J, \mathbb{R}^n) \to C^0(J, \mathbb{R}^m)$ by $A(\phi)(t) = B(t)\phi(t)$. Show that A is linear and bounded.

2. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be continuous. Let J be an interval. Define a map $F : C^0(J, \mathbb{R}^n) \to C^0(J, \mathbb{R}^m)$ by $F(\phi)(t) = f(\phi(t))$. Show that F is continuous. Suggestion: Let $\phi \in C^0(J, \mathbb{R}^n)$. Then the range of ϕ is bounded. Let W be a large ball in $\mathbb{R}^n$ that contains the range of ϕ. Then f is uniformly continuous on W. This should help you to show that F is continuous at ϕ.

3. Define $F : C^0([a,b], \mathbb{R}) \to \mathbb{R}$ by $F(\phi) = \int_a^b (\phi(t))^2 \, dx$. Using the definition of derivative, prove that F is differentiable with $DF(\phi)\psi = \int_a^b 2\phi(t)\psi(t)\, dt$. Then use this formula to show that F is C^1.