MA 732 Homework 10

S. Schecter

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1. A bead on a rotating hoop satisfies the differential equation

$$\ddot{x} + \dot{x} + \sin x - \mu \sin 2x = 0.$$

Here x is measured in radians from the bottom of the hoop, and the parameter μ is related to the spin rate of the hoop. Letting $y = \dot{x}$, we obtain the system

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\sin x + \mu \sin 2x - y. \end{aligned}$$

(a) Show that for every μ, $(x, y) = (0, 0)$ is an equilibrium.

(b) Show that the equilibrium at $(x, y) = (0, 0)$ is attracting for $\mu < \frac{1}{2}$, has a 0 eigenvalue for $\mu = \frac{1}{2}$, and is not attracting for $\mu > \frac{1}{2}$.

(c) Use center manifold reduction to show that a pitchfork bifurcation occurs at $\mu = \frac{1}{2}$. Suggestions:

- Let $\lambda = \mu - \frac{1}{2}$.
- Let $y = h(x, \lambda) = x(A + Bx + C\lambda + Dx^2 + \ldots)$. No more terms should be needed.
- Recall that $\sin x = x - \frac{x^3}{3!} + \ldots$.

(d) Are the new equilibria that appear in the pitchfork bifurcation attracting? (Suggestion: begin by looking at the bifurcation diagram.)