MA 532 Test 1

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1. Consider the differential equation $\frac{dx}{dt} = (a - x)(b - x)$ with $0 < a < b$.

   (a) Show that by a change of coordinates of the form $x = mu$, $t = qs$, with $m > 0$ and $q > 0$, one can convert this differential equation into the form $\frac{du}{ds} = (1 - u)(c - u)$ with $c > 1$. Give formulas for m, q, and c in terms of a and b.

   (b) Sketch the phase portrait of $\frac{dx}{dt} = (1 - x)(2 - x)$. Be sure to show equilibria.

2. Sketch the phase portrait of $\ddot{x} - x - 3x^2 = 0$. Don't forget to show equilibria.

3. Consider the system

$$\dot{x} = x(2 - x - y),$$
$$\dot{y} = y(1 - x)$$

   Draw the nullclines; show the equilibria; draw representative vectors, including vectors on the nullclines; and draw some typical solution curves. You only need to consider the region $x \geq 0$ and $y \geq 0$.

4. The differential equation $\dot{x} = Ax$, with A a 2×2 matrix, has the eigenvalues -1 and -2. An eigenvector for -1 is $(1, 2)$; an eigenvector for -2 is $(2, 1)$. Draw the phase portrait.

5. Compute e^{At} for

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}.$$

   You may leave your answer as a product of three matrices. Hint: there is a repeated eigenvalue.