MA 440-001
Test 1 Answers

Thanks to:
Alex Chin

1a

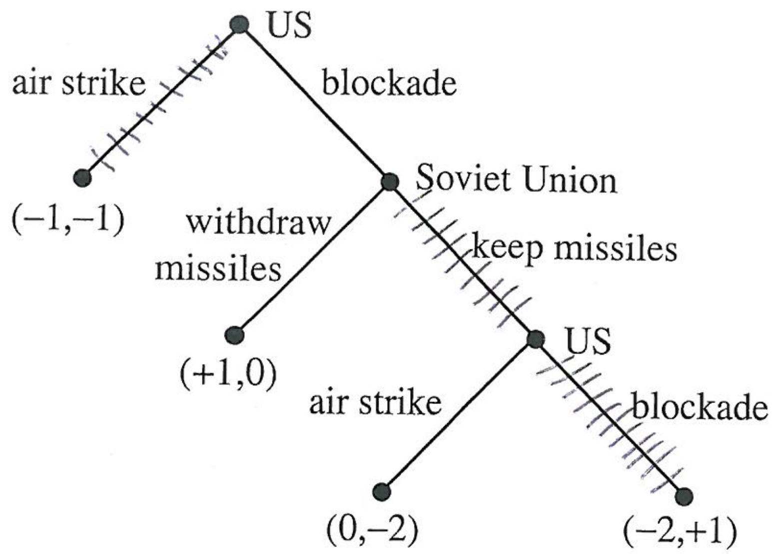


Figure 1: Payoffs to U.S. are shown first, payoffs to Soviet Union are shown second.

US blockades, SU withdraws missiles.

b. Four US strategies:

15 AA: Airstrike at top node, Airstrike at other US node

AB: " " " ", Blockade " " " "

BA Blockade " " ", Airstrike " " " "

BB " " " ", Blockade " " " "

Two Soviet strategies:

W Withdraw missiles if US blockades

K Keep " " " "

SU

A.	W	K
US AA	(-1,-1)	(-1,-1)
AB	(-1,-1)	(-1,-1)
BA	(1,0)	(0,-2)
BB	(1,0)	(-2,1)

30

2. Consider $\pi_2(x,y) = (1-x)(4y-4xy-y^2)$

$$\frac{\partial \pi_2}{\partial y} = (1-x)(4-4x-2y) = 0$$

$\Rightarrow x=1$ or $(4-4x-2y=0 \Rightarrow y = 2-2x)$.

We assume $x \neq 1$ so $y = 2-2x$. *

Now consider

$$\pi_1(x, 2-2x) = x\left(4(2-2x) - 4x(2-2x) - (2-2x)^2\right)$$
$$= x\left(8-8x-8x+8x^2-4+8x-4x^2\right)$$
$$= 4x^3 - 8x^2 + 4x.$$

So $\frac{d\pi_1}{dx} = 12x^2 - 16x + 4 = 0$

$\Rightarrow 3x^2 - 4x + 1 = 0$

$\Rightarrow (x-1)(3x-1) = 0$

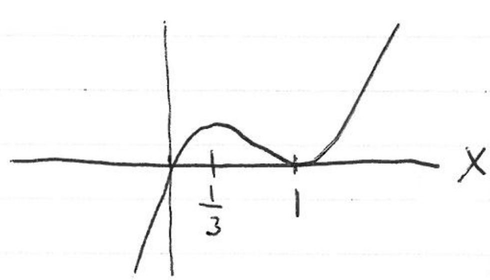
$\Rightarrow x = 1$ or $x = \frac{1}{3}$

We know $x \neq 1$, so the government sets a tax rate of $x = \frac{1}{3}$. **

* This is a max because for fixed x, $0 \le x < 1$, $(1-x)(4y-4xy-y^2)$, as a function of y, is a parabola opening downward.

** Graph of the function $4x^3 - 8x^2 + 4x$:

Max for $0 \le x \le 1$ occurs at $x = \frac{1}{3}$.



Alex Chin

3. a. There are no Nash equilibria where the total of the three requests is less than \$6. In this case, none of the daughters is asking for \$4. Then any one of the daughters could increase her request by \$1 and her payoff will go up, while the total remains ≤ \$6.

10 ✓

b. Every strategy profile where the total is \$6 is a Nash equilibrium. If any daughter increases her request, the total exceeds \$6 and her payoff drops to nothing. If she lowers her request, then her payoff will drop by that amount.

10 ✓

c. The situations where the requests total more than \$6 but there exists a pairwise sum less than \$6, e.g. \$4, \$3, \$1, are not Nash equilibria because one of the daughters has the opportunity to lower her request (e.g. from \$4 to \$2) to increase her payoff from 0 to a positive number. The situations where every pairwise sum is greater than or equal to \$6, e.g. \$4, \$2, \$4, are Nash equilibria because no matter how much any player drops her request, the total will still be larger than \$6 and her payoff will not improve.

10 ✓ ✓

d. "Ask for 1 dollar" does not weakly dominate "ask for 4 dollars." If the other two daughters both ask for \$1, then Daughter 1 gets a payoff of \$1 if she asks for 1 dollar and \$4 if she asks for \$4 (even though asking for 4 dollars is never better off for every other strategy combination for Daughters 2 and 3).

10 ✓