MA 440 Test 1

S. Schecter

September 23, 2011

1. Here is another model of the Cuban Missile Crisis of 1962. When the U.S. discovered that the Soviet Union had installed missiles with nuclear warheads in Cuba, it seriously considered only two options, naval blockade and air strike. If the U.S. blockaded, the Soviet Union would have to decide whether to keep its missiles in Cuba or withdraw them. If it decided to keep the missiles in Cuba, the U.S. would have to decide whether to continue the blockade or launch an air strike.

   Payoffs to the U.S. (Player 1):

   - $+1$ if the Soviet Union withdraws the missiles without an air strike.
   - 0 if the U.S. launches an air strike after having shown restraint. (The air strike will eliminate the missiles, but it is dangerous because it could lead to war.)
   - -1 if the U.S. launches an immediate air strike. (The air strike will eliminate the missiles, but it is dangerous because it could lead to war. Also, if the U.S. does not show restraint, the Soviet Union will win a public relations victory.)
   - -2 if the Soviet Union keeps its missiles in Cuba.

   Payoffs to the Soviet Union (Player 2):

   - $+1$ if it keeps its missiles in Cuba.
   - 0 if it withdraws its missiles without an air strike. (At least there is no danger of war.)
   - -1 if it withdraws its missiles after an immediate air strike. (There is a danger of war, which is very bad, but at least the Soviet Union will win a public relations victory because the U.S. showed no restraint.)
   - -2 if it withdraws its missiles after a U.S. airstrike that follows a period of U.S. restraint. (There is a danger of war, which is very

bad, and the Soviet Union does not even win a public relations victory.)

The following tree diagram shows the situation.

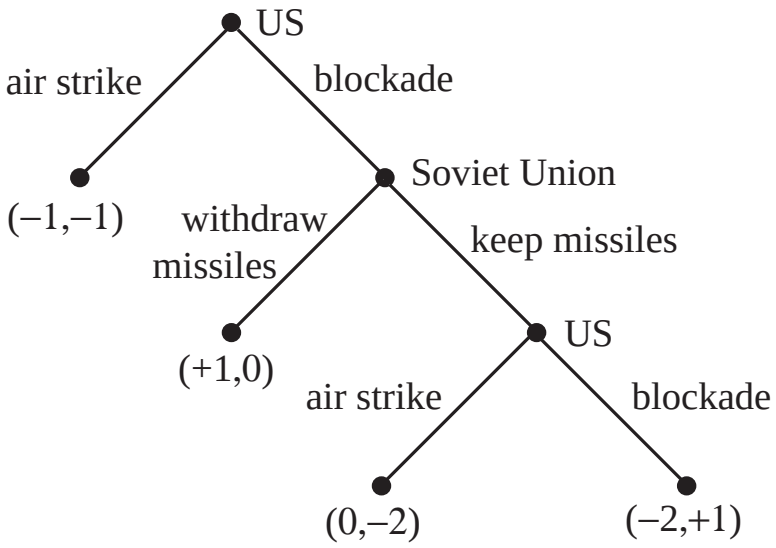

Figure 1: Payoffs to U.S. are shown first, payoffs to Soviet Union are shown second.

(a) Use backward induction to figure out what the U.S. should do and what the outcome will be. Be sure I can follow your reasoning.

(b) Make a normal form representation of this game (i.e., a matrix that shows each player's strategies and the resulting payoffs).

2. Government chooses a tax rate x, $0 \leq x \leq 1$. Citizens then choose a level of effort y to devote to making money. The resulting level of economic activity a, in trillions of dollars, is

$$a = 4y - 4xy - y^2$$

Explanation: The term $4y$ expresses the idea that economic activity should be proportional to effort. There are two correction terms. The term $-4xy$ expresses the idea that when the tax rate is high, much effort goes into avoiding taxes, not into economic activity. The term $-y^2$ expresses the idea that too much effort is counterproductive.

The government's payoff is the taxes it collects, computed as tax rate x times level of economic activity a:

$$\pi_1(x, y) = xa = x(4y - 4xy - y^2).$$

The citizens' payoff is the part of economic activity that they keep after taxes:

$$\pi_2(x, y) = a - xa = (1 - x)a = (1 - x)(4y - 4xy - y^2).$$

We regard this as a two-player game. Player 1 is the government. Player 2 is the citizens. Player 1 chooses the tax rate x, then Player 2 observes x and chooses y. The payoffs are given above.

Use backward induction to find the tax rate x that maximizes the government's payoff.

3. A mother has three daughters. She tells each to ask for a whole number of dollars between 1 and 4 (inclusive) as a Christmas present. If the total of the three requests is less than or equal to \$6, each daughter will receive what she asked for. If the total of the three requests is greater than \$6, each daughter will receive nothing. The three daughters make their requests simultaneously and independently.

   (a) Find all Nash equilibria in which the total of the three requests is less than \$6. Explain your answer briefly.

   (b) Find all Nash equilibria in which the total of the three requests is exactly \$6. Explain your answer briefly.

   (c) Find all Nash equilibria in which the total of the three requests is greater than \$6. Explain your answer briefly. Be careful with this part!

   (d) Does Daughter 1's strategy "ask for 1 dollar" weakly dominate her strategy "ask for 4 dollars"? Explain.

   For parts (a)–(c), you don't have to list all the Nash equilibria, you can just describe in words which strategy profiles are Nash equilibria and which are not, and explain.