MA 493 B

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Test 2 Answers

① a)

		q w	$1-q$ e
p	w	(2,2)	(2,3)
$1-p$	e	(3,2)	(0,0)

$$2q + 2(1-q) = 3q + 0(1-q)$$
$$2 = 3q$$
$$q = \frac{2}{3}$$
Similarly, $p = \frac{2}{3}$

Both watch with prob. $\frac{2}{3}$, eat with prob. $\frac{1}{3}$

b) Let $\sigma = p\,w + (1-p)e$ with $p = \frac{2}{3}$.

Let $\tau = q\,w + (1-q)e$

$\pi_{\sigma\sigma} = \pi_{\tau\sigma}$ for every τ.

$$\pi_{\sigma\tau} - \pi_{\tau\tau} = pq\cdot 2 + p(1-q)\cdot 2 + (1-p)q\cdot 3 + (1-p)(1-q)\cdot 0$$
$$-\left[qq\cdot 2 + q(1-q)\cdot 2 + (1-q)q\cdot 3 + (1-q)(1-q)\cdot 0\right]$$

$$= (p-q)(2q + 2(1-q) - 3q)$$
$$= (p-q)(2-3q) = 3(p-q)(\tfrac{2}{3}-q) = 3(\tfrac{2}{3}-q)^2 > 0$$

for $q \neq \frac{2}{3}$.

Therefore σ is evolutionarily stable.

② a)

		q_1 w	q_2 e	q_3 m
p_1	w	(1,1)	(0,2)	(0,2)
p_2	e	(2,0)	(-1,-1)	(2,0)
p_3	m	(2,0)	(0,2)	(-1,-1)

The following are equal: $\begin{cases} q_1 \cdot 1 + q_2 \cdot 0 + q_3 \cdot 0 \\ q_1 \cdot 2 + q_2 \cdot -1 + q_3 \cdot 2 \\ q_1 \cdot 2 + q_2 \cdot 0 + q_3 \cdot -1 \end{cases}$

$$\Rightarrow \begin{cases} q_1 = 2q_1 - q_2 + 2q_3 \\ q_1 = 2q_1 - q_3 \end{cases} \Rightarrow \begin{cases} q_1 - q_2 + 2q_3 = 0 \\ q_1 \qquad - q_3 = 0 \end{cases}$$

Also: $q_1 + q_2 + q_3 = 1$

$$\Rightarrow q_1 = q_3 = \frac{1}{5}, \quad q_2 = \frac{3}{5}$$

Both birds use w and m $\frac{1}{5}$ of the time, e $\frac{3}{5}$ of the time.

b)

		q w	e	$1-q$ m
	w	(1,1)	(0,2)	(0,2)
p	e	(2,0)	(-1,-1)	(2,0)
$1-p$	m	(2,0)	(0,2)	(-1,-1)

$$q \cdot -1 + (1-q) \cdot 2 = q \cdot 0 + (1-q) \cdot -1$$
$$-q + 2 - 2q = -1 + q$$
$$3 = 4q$$
$$q = \frac{3}{4}$$

Both birds use $\sigma = \frac{3}{4} e + \frac{1}{4} m$

$\pi_1(e, \sigma) = \pi_1(m, \sigma) = -\frac{1}{4}$

$\pi_1(w, \sigma) = 0$

Since $0 > -\frac{1}{4}$, (σ, σ) is <u>not</u> a Nash equilibrium.

(3) σ^* = Worker's trigger strategy

τ^* = Boss's trigger strategy

Let σ be another strategy for Worker.

(1) If Worker always works hard, payoffs are unchanged.

(2) If Worker first slacks off in round k,

Worker's payoff from round k on is at most $\Big\}$ $5\delta^k + 3\delta^{k+1} + 3\delta^{k+2} + \cdots = 5\delta^k + 3\delta^{k+1} \cdot \frac{1}{1-\delta}$

If Worker had continued to use σ^*, payoff from round k on would be $\Big\}$ $4\delta^k + 4\delta^{k+1} + \cdots = 4\delta^k \cdot \frac{1}{1-\delta}$

$$4\delta^k \cdot \frac{1}{1-\delta} \geq 5\delta^k + 3\delta^{k+1} \cdot \frac{1}{1-\delta} \iff$$

$$\frac{4}{1-\delta} \geq 5 + 3\delta \cdot \frac{1}{1-\delta} \iff 4 \geq 5(1-\delta) + 3\delta \iff$$

$$2\delta \geq 1 \iff \delta \geq \frac{1}{2}$$

Let τ be another strategy for Boss.

(1) If Boss always pays H, payoffs are unchanged

(2) If Boss first pays L in round k,

Boss's payoff from round k on is at most $\Big\}$ $4\delta^k + 0\delta^{k+1} + 0\delta^{k+2} + \cdots = 4\delta^k$

If Boss had continued to use σ^*, payoff from round k on would be $\Big\}$ $2\delta^k + 2\delta^{k+1} + \cdots = 2\delta^k \cdot \frac{1}{1-\delta}$

$$2\delta^k \cdot \frac{1}{1-\delta} \geq 4\delta^k \iff \frac{2}{1-\delta} \geq 4 \iff \underline{\underline{\delta \geq \frac{1}{2}}}.$$

So (σ^*, τ^*) is a Nash equilibrium for $\delta \geq \frac{1}{2}$.

(4) a) Tough Guy always gets payoff of 2 from Steak and -2 from Quiche.

b) Wimp's payoffs:

		HH	HL	LH	LL
Wimp	S	-8	-8	-2	-2
	Q	-4	2	-4	2

$(*)$

Bully encounters Tough Guy $\frac{1}{2}$ the time. His payoffs are

		HH	HL	LH	LL
Wimp	S	-2	-2	0	0
	Q	-2	-2	0	0

(Reason: In this case, Wimp's strategy is irrelevant, and Tough Guy always eats steak.)

Bully encounters Wimp $\frac{1}{2}$ the time. His payoffs are

		HH	HL	LH	LL
Wimp	S	2	2	0	0
	Q	2	0	2	0

Bully's payoffs are $\frac{1}{2}$ first matrix + $\frac{1}{2}$ second matrix:

		HH	HL	LH	LL
Wimp	S	0	0	0	0
	Q	0	-1	1	0

(**)

Answer from (*) and (**):

		Bully HH	HL	LH	LL
Wimp	S	(-8,0)	(-8,0)	(-2,0)	(-2,0)
	Q	(-4,0)	(2,-1)	(-4,1)	(2,0)