MA 493B Test 2

S. Schecter

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1. When certain birds find a recently deceased small animal, they eat it. If two birds eat the animal together, a crow may surprise them and drive them off. However, if at least one bird stays watchful, it can scare off any crows that may happen by. The watchful bird gets less to eat, and incurs an energy cost.

   Each bird has two strategies, watch or eat. The payoffs are as follows.

		Bird 2	
		watch	**eat**
Bird 1	**watch**	$(2, 2)$	$(2, 3)$
	eat	$(3, 2)$	$(0, 0)$

   (a) Find a mixed strategy Nash equilibrium in which both birds use both strategies with positive probability. (Because of the symmetry of the problem, once you have found the probabilities for one bird, you may assume that the other bird uses the same probabilities.)

   (b) Check whether your mixed strategy Nash equilibrium is evolutionarily stable.

2. Let's complicate the previous problem by assuming that the birds have three strategies:

   - watch (w)
   - eat (e)
   - mixed (m): watch when you sense that the other bird is a greedy eater who never watches; otherwise eat.

   This time we'll assume the payoffs are as follows:

		Bird 2		
		w	**e**	**m**
	w	$(1,1)$	$(0,2)$	$(0,2)$
Bird 1	**e**	$(2,0)$	$(-1,-1)$	$(2,0)$
	m	$(2,0)$	$(0,2)$	$(-1,-1)$

   (a) Find a mixed strategy Nash equilibrium in which both birds use all three strategies with positive probability. (Because of the symmetry of the problem, once you have found the three probabilities for one bird, you may assume that the other bird uses the same three probabilities.)

   (b) Try to find a Nash equilibrium in which both birds use strategies e and m with positive probability, and strategy w with 0 probability. (Again, because of the symmetry of this problem, once you have found the two probabilities for one bird, you may assume that the other bird uses the same two probabilities.) Don't forget the final step in checking that you really have a Nash equilibrium.

3. A boss can pay a worker \$4K per week (High) or \$2K per week (Low). The worker can Work Hard or Slack Off. If the worker Works Hard, he earns \$6K for his boss. If the worker Slacks Off, he earns only \$2K for his boss. In addition, if the worker Slacks Off, he has more time to spend napping, reading comic books, and surfing the internet. This is worth \$1K to the worker.

   The payoff matrix of this game is

		Boss	
		High	**Low**
Worker	**Work Hard**	$(4, 2)$	$(2, 4)$
	Slack Off	$(5, -2)$	$(3, 0)$

   The game is repeated every week.

   Suppose the boss and the worker both use trigger strategies:

   - The boss starts by paying High. If the worker Works Hard in period k, the boss pays High in period $k + 1$. If the worker Slacks Off in period k, the boss pays Low in period $k + 1$ and in every subsequent period.
   - The worker starts by Working Hard. If the boss pays High in period k, the worker Works Hard in period $k + 1$. If the boss pays Low in period k, the worker Slacks Off in period $k+1$ and in every subsequent period.

   The discount factor is δ, $0 < \delta < 1$.

   Find a number δ_0 such that if $\delta > \delta_0$, it is a Nash equilibrium for both boss and worker to use these trigger strategies.

4. A Bully hangs out at a bar that has two types of customers, Tough Guys and Wimps. A customer whom the Bully does not know walks into the bar, sits down, and orders either Steak (which Tough Guys love and Wimps hate) or Quiche (which Tough Guys hate and Wimps love). The Bully may harass the customer or leave him alone. Harassment does not bother Tough Guys, who will just throw the Bully out of the bar; but it bothers Wimps very much. Everyone knows the following:

   - Bully's payoffs
     - Leave customer alone: 0.
     - Harass Tough Guy: -2.
     - Harass Wimp: $+2$.
   - Tough Guy's payoffs
     - Eat steak: $+2$.
     - Eat quiche: -2.
     - Get harassed by Bully: 0.
     - Get left alone by Bully: 0.
   - Wimp's payoffs
     - Eat steak: -2.
     - Eat quiche: $+2$.
     - Get harassed by Bully: -6.
     - Get left alone by Bully: 0.

   Half the customers are Tough Guys and half are Wimps.

   The game tree on the next page illustrates the situation.

   There are three players, Tough Guy, Wimp, and Bully. The Bully does not know which type the customer is. He only knows what the customer orders.

   The customers each have two strategies: Steak (S) and Quiche (Q). The bully has four strategies:

   - HH: Harass the customer whatever he orders
   - HL: Harass customers who order Steak. Leave alone customers who order Quiche.
   - LH: Leave alone customers who order Steak. Harass customers who order Quiche.
   - LL: Leave the customer alone whatever he orders.

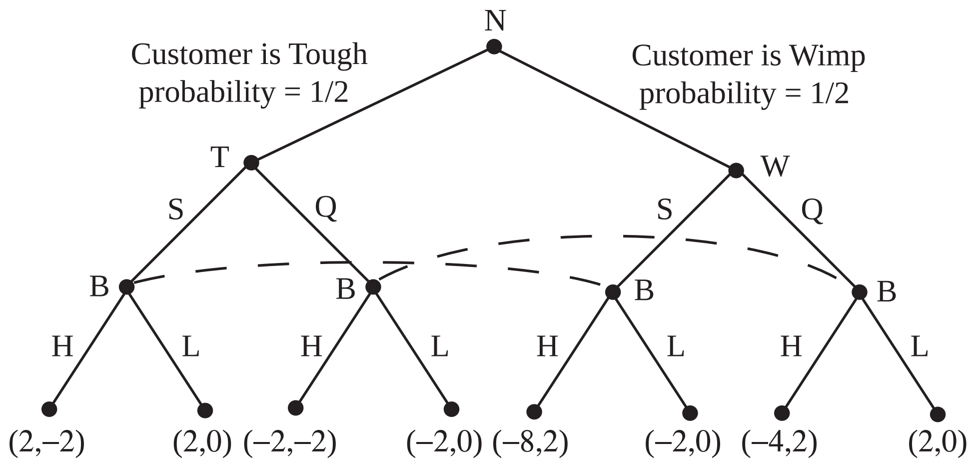

Figure 1: N = Nature, T = Tough Guy, W = Wimp, S = Steak, Q = Quiche, B = Bully, H = Harass customer, L = Leave customer alone. The first payoff is to the customer (whether Tough or Wimp), the second is to the Bully.

(a) Explain why the Tough Guy will always order Steak. (Note that backward induction is not appropriate for this problem, since the Bully does not know the type of the customer.)

(b) Assume that the Tough Guy always orders steak. Complete the following 2×4 payoff matrix, showing only payoffs to the Wimp and the Bully.

		Bully HH	HL	LH	LL
Wimp	**Steak**				
	Quiche				

Be careful in calculating the Bully's payoffs! Once the Bully chooses a strategy, half the time he uses it against a Tough Guy, who always orders Steak; half the time he uses it against a Wimp.