

# MA 493B Test 2

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1. **Chicken with Three Strategies.** Two drivers drive toward each other. Each has three strategies: swerve left ( $L$ ), drive straight ( $S$ ), swerve right ( $R$ ). If one driver drives straight while the other swerves, the first driver's reputation increases, and the second driver's decreases. However, if both drive straight, or if one swerves left and the other swerves right, there is a crash, and both are injured. The payoffs are as follows:

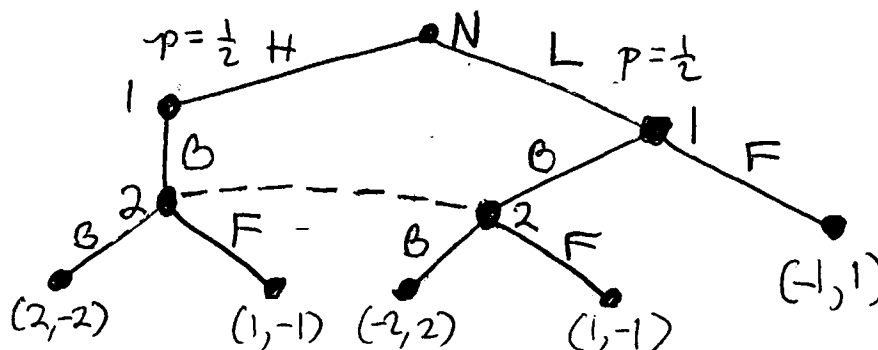
		Driver 2		
		L	S	R
Driver 1	L	(0,0)	(-1,1)	(-2,-2)
	S	(1,-1)	(-2,-2)	(1,-1)
	R	(-2,-2)	(-1,1)	(0,0)

- (a) Find a Nash equilibrium in which both drivers use all three strategies with positive probability. (Because of the symmetry of the problem, once you have found the three probabilities for one driver, you may assume that the other driver uses the same three probabilities.)
- (b) Find a Nash equilibrium in which both drivers use strategies  $L$  and  $S$  with positive probability, and strategy  $R$  with 0 probability. (Again, because of the symmetry of this problem, once you have found the the two probabilities for one driver, you may assume that the other driver uses the same two probabilities.) Don't forget the final step in checking that you really have a Nash equilibrium.

2. One-Card One-Round Poker. There are two player, and there is a deck of two cards, one high ( $H$ ) and one low ( $L$ ).

1. Both players put \$1 into the pot.
2. Player 1 picks a card randomly from the deck and looks at it. He does not show it to Player 2.
3. Player 1 either folds or bets \$1.
4. If Player 1 folds, Player 2 wins the pot. His gain is \$1.
5. If Player 1 bets, Player 2 either folds or bets \$1.
6. If Player 2 folds, Player 1 wins the pot. His gain is \$1.
7. If Player 2 bets, both players look at Player 1's card. If it is  $H$ , Player 1 wins the pot; his gain is \$2. If it is  $L$ , Player 2 wins the pot; his gain is \$2.

We model this game as a tree in which the first move is Nature's: Nature decides whether the card drawn is high or low, each with probability  $1/2$ . We will assume that if Player 1 is dealt the high card, he always bets. (If he folds, he loses \$1; if he bets, he will gain at least \$1.)



Thus Player 1 has only two pure strategies:

BB If dealt the high card, bet; if dealt the low card, bet.

BF If dealt the high card, bet; if dealt the low card, fold.

Player 2 also has two pure strategies:

B If Player 1 bets, bet.

F If Player 1 bets, fold.

(a) Find the payoff matrix for this game.

(b) Check whether there are any pure-strategy Nash equilibria.

3. **Smallville Bar.** The town of Smallville has three residents. At night, each has two choices: watch TV ( $T$ ) or walk to the bar ( $B$ ). The energy cost of watching TV is 0, and the utility is also 0. The energy cost of walking to the bar is 1; the utility is 0 if no one else is at the bar, 2 if one other resident is at the bar, and 1 if both other residents are at the bar. (The residents of Smallville are sociable, but not too sociable.) The payoffs are therefore as follows:

Resident 3 uses strategy T

		Resident 2	
		T	B
Resident 1	T	(0,0,0)	(0,-1,0)
	B	(-1,0,0)	(1,1,0)

Resident 3 uses strategy B

		Resident 2	
		T	B
Resident 1	T	(0,0,-1)	(0,1,1)
	B	(1,0,1)	(0,0,0)

Suppose Resident 1 uses the mixed strategy  $xT + (1 - x)B$ , Resident 2 uses the mixed strategy  $yT + (1 - y)B$ , and Resident 3 uses the mixed strategy  $zT + (1 - z)B$ .

- (a) Give Player 1's payoff function  $\Pi_1(x, y, z)$ . Don't multiply out.  
 (b) Show that  $(x, y, z) = (\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$  is a mixed-strategy Nash equilibrium.

Here is one way to do this: Calculate  $\frac{\partial \Pi_1}{\partial x}$ . From the symmetry of the problem,  $\frac{\partial \Pi_2}{\partial y}$  and  $\frac{\partial \Pi_3}{\partial z}$  are given by the same formula with the roles of the variables changed. Check that  $(x, y, z) = (\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$  satisfies the three equations

$$\frac{\partial \Pi_1}{\partial x} = 0, \quad \frac{\partial \Pi_2}{\partial y} = 0, \quad \frac{\partial \Pi_3}{\partial z} = 0.$$

## Test 2 Answers

20pts

① a)

		$q_1$	$q_2$	$q_3$
		L	S	R
$p_1$	L	(0,0)	(-1,1)	(-2,-2)
$p_2$	S	(1,-1)	(-2,-2)	(1,-1)
$p_3$	R	(-2,-2)	(-1,1)	(0,0)

If driver 2 uses all 3 strategies, the following are equal

$$(1) p_1 \cdot 0 + p_2 \cdot 1 + p_3 \cdot -2$$

$$(2) p_1 \cdot 1 + p_2 \cdot -2 + p_3 \cdot 0$$

$$(3) p_1 \cdot -2 + p_2 \cdot -1 + p_3 \cdot 0$$

Setting (1) = (3) and (2) = (3) gives

$$2p_1 - 2p_3 = 0$$

$$3p_1 - p_2 + p_3 = 0$$

Also,

$$p_1 + p_2 + p_3 = 1$$

First equation says  $p_3 = p_1$ . Substitute into last two:

$$4p_1 - p_2 = 0$$

$$2p_1 + p_2 = 1$$

Solution is  $p_1 = \frac{1}{6}$ ,  $p_2 = \frac{2}{3}$ . Since  $p_3 = p_1$ ,  $p_3 = \frac{1}{6}$ .

By symmetry,

$$q_1 = \frac{1}{6}, \quad q_2 = \frac{2}{3}, \quad q_3 = \frac{1}{6}$$

13pts

b) Equations are

$$p_1 \cdot 0 + p_2 \cdot -1 = p_1 \cdot 1 + p_2 \cdot -2$$

$$p_1 + p_2 = 1$$

or

$$-p_1 + p_2 = 0$$

$$p_1 + p_2 = 1$$

Solution is  $p_1 = \frac{1}{2}$ ,  $p_2 = \frac{1}{2}$ , and of course  $p_3 = 0$

By symmetry,  $q_1 = \frac{1}{2}$ ,  $q_2 = \frac{1}{2}$ ,  $q_3 = 0$

We of course have  $\Pi_2 \left( \left( \frac{1}{2}, \frac{1}{2}, 0 \right), L \right) = \Pi_2 \left( \left( \frac{1}{2}, \frac{1}{2}, 0 \right), S \right)$ .

5pts For a Nash equilibrium, this number must be  $\geq \Pi_2 \left( \left( \frac{1}{2}, \frac{1}{2}, 0 \right), R \right)$ .

$$\Pi_2 \left( \left( \frac{1}{2}, \frac{1}{2}, 0 \right), L \right) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot -1 + 0 \cdot -2 = -\frac{1}{2}$$

$$\Pi_2 \left( \left( \frac{1}{2}, \frac{1}{2}, 0 \right), R \right) = \frac{1}{2} \cdot -2 + \frac{1}{2} \cdot -1 + 0 \cdot 0 = -\frac{3}{2}$$

25pts

② a) IP H:

	B	F
BB	(2,2)	(1,-1)
BF	(2,-2)	(1,-1)

IP L:

	B	F
BB	(-2,2)	(1,-1)
BF	(-1,1)	(-1,1)

$\frac{1}{2}$  Matrix 1 +  $\frac{1}{2}$  Matrix 2 =

	B	F
BB	(0,0)	(1,-1)
BF	( $\frac{1}{2}, \frac{1}{2}$ )	(0,0)

5pts

b)

	B	F
BB	(0,0)	(1,-1)
BF	( $\frac{1}{2}, \frac{1}{2}$ )	(0,0)

No pure-strategy Nash equilibria.

6 pts ③ a)  $\Pi_1(x, y, z) = (1-x)yz \cdot -1 + (1-x)(1-y)z \cdot 1$   
 $+ (1-x)y(1-z) \cdot 1$

6 pts b)  $\frac{\partial \Pi_1}{\partial x} = yz - (1-y)z - y(1-z)$   
 $= yz - z + yz - y + yz$   
 $= 3yz - y - z$

By symmetry  $\frac{\partial \Pi_2}{\partial y} = 3xz - x - z$

$\frac{\partial \Pi_3}{\partial z} = 3xy - x - y$

Our equations are

$$3yz - y - z = 0$$

$$3xz - x - z = 0$$

$$3xy - x - y = 0$$

Set  $x=y=z=\frac{2}{3}$  in each:  $3 \cdot \frac{2}{3} \cdot \frac{2}{3} - \frac{2}{3} - \frac{2}{3} = 0$  ✓

Remark  $\Pi_1\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right) = \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot -1 + \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot 1$

$+ \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot 1 = 0$ . Similarly,  $\Pi_2\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right) = \Pi_3\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right) = 0$ .

The payoffs are the same as in the Nash equilibrium where all ~~do~~ watch TV.