MA 426/591M Test 1

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1. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. For each of the following questions, if you answer "yes", give a brief explanation; if you answer "no", give an example that shows "no" is the correct answer.

   (a) Is $\{x \in \mathbb{R} : f(x) = 0\}$ necessarily a closed set?

   (b) Is $\{x \in \mathbb{R} : f(x) = 0\}$ necessarily a compact set?

   (c) Is $\{x \in \mathbb{R} : f(x) = 0\}$ necessarily a path connected set?

   (d) Is $f([0, \infty))$ necessarily a closed set?

   (e) Is $f([0, 1])$ necessarily a closed set?

   (f) Is $f([0, \infty))$ necessarily a path connected set?

Do *four* of the following five problems. *Do not do all five problems.* If you do, I'll just grade the first four that you attempt.

2. Let $A = \{x_1, x_2, \dots, x_N\}$ be a finite set of points in $\mathbb{R}^n$. Prove that A is closed by showing that the complement is open.

3. Let A be a subset of $\mathbb{R}^n$. Let B be the set of all accumulation points of A. Prove that B is closed. Suggestion: Let x be an accumulation point of B. Show that $x \in B$.

4. Assume that $f : \mathbb{R}^n \to \mathbb{R}^m$ is continuous, *i.e.*, f satisfies the ϵ-δ definition of continuity at every point of $\mathbb{R}^n$. Let $x_k \to x$ in $\mathbb{R}^n$. Prove: $f(x_k) \to f(x)$ in $\mathbb{R}^m$.

5. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a function. Assume that for every open set $U \subset R^m$, $f^{-1}(U)$ is open. Prove that f is continuous.

6. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be continuous and let $K \subset \mathbb{R}^n$ be compact. Prove that $f(K)$ is compact by showing that every open cover of $f(K)$ has a finite subcover.