Material to Review for the Second MA 426 Test

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In the following, when I say you should know a definition or the statement of something, that doesn't mean you should know it word-for-word; it just means that this is something that you should be able to use. When I say you should know a proof, again I don't mean word-for-word, but that you should be able to prove this thing if asked. The problems listed are ones you should be able to do.

Whenever the book mentions metric spaces in definitions, theorems, etc., you can substitute \mathbb{R}^n .

- 5.1. Pointwise and uniform convergence. Definitions 5.1.1 and 5.1.2; the lemma proved in lecture for showing that a sequence f_k converges uniformly; proof of Proposition 5.1.4. Homework assigned March 3, problems 1 and 2.
- 6.1. Definition of the derivative. Definition 6.1.1. Proof that if f(x) = Ax + b then Df(x) = A.
- 6.2. Matrix representation. Definition 6.2.1. Proof of Theorem 6.2.2.
- 6.3. Continuity of differentiable functions. Proofs of the following: (1) if $L : \mathbb{R}^n \to \mathbb{R}^m$ is linear, then there is a constant M such that $|| L(x) || \leq M || x ||$; (2) if $f : A \to \mathbb{R}^m$ is differentiable at x_0 , then f is continuous at x_0 .
- 6.4. Conditions for differentiability. The example that begins the section (figure 6.4.1). Statement of Theorem 6.4.1. Definition 6.4.2. Example 6.4.3. Homework assigned March 21, problems 1, 2.
- 6.5. Differentiation rules. Proof of the "multiplication by a constant rule" and the "sum rule." Chain rule. Relation of the chain rule to the directional derivative formula (Example 6.5.4 as done in lecture). Homework assigned March 21, problem 4. P. 386, problems 13ac.
- 6.7. Mean value theorem. Proof of Theorem 6.7.1 (first part).

- 6.8. Second derivative and Taylor's Theorem. The idea of the derivative of *Df*. Statements of Theorems 6.8.2 and 6.8.3. The version of Theorem 6.8.5 that we did in lecture, and its proof. Homework assigned March 21, problem 3. Problem 6.8.5. P. 386, problem 16.
- 6.9 Maxima and minima. Statement of Theorem 6.9.2. Proofs of Theorem 6.9.4, and of the lemma we used to prove it. Problem 6.9.6.
- 7.1. Inverse Function Theorem. Statements of Contraction Mapping Theorem and Theorem 7.1.1. Problems 7.1.1, 7.1.2, 7.1.5, and p.442 problem 25.
- 7.2. Implicit Function Theorem. Statement of Theorem 7.2.1. Pp. 439–440 problems 4, 6, 8.
- 7.7. Lagrange Multipliers. Omit.