Material to Review for the First MA 426/591M Test

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February 14, 2003

In the following, when I say you should know a definition or the statement of something, that doesn't mean you should know it word-for-word; it just means that this is something that you should be able to use. When I say you should know a proof, again I don't mean word-for-word, but that you should be able to prove this thing if asked. The problems listed are ones you should be able to do.

Whenever the book mentions metric spaces in definitions, theorems, etc., you can substitute $\mathbb{R}^n$.

- 1.6 and 1.7. Euclidean space, norms, inner products, metrics. Consider these sections as background.
- 2.1. Open sets. Definition 2.1.1; proof of Proposition 2.1.2; proofs that $\{x \in \mathbb{R}^n : \| x \| > 1\}$ and $\{x \in \mathbb{R}^2 : x_1 > 0\}$ are open; proof of Proposition 2.1.3; homework assigned Jan. 17, problems 1, 2, 3.
- 2.2. Interior of a set. Definition 2.2.1; homework assigned Jan. 17, problems 4, 5, 6.
- 2.3. Closed sets. Definition 2.3.1, statement of Proposition 2.3.2. Proofs from lecture that various sets are closed.
- 2.4. Accumulation points. Definition 2.4.1, statement of Theorem 2.4.2; homework assigned Jan. 24, problem 1.
- 2.5. Closure. Definition 2.5.1, statement of Proposition 2.5.2, description of closure in terms of ϵ-neighborhoods from lecture.
- 2.6. Boundary. Definition 2.6.1, statement of Proposition 2.6.2; homework assigned Jan. 24, problems 2, 3.

- 2.7. Sequences. Definition 2.7.1, statement of Propositions 2.7.2 and 2.7.3, proof of Proposition 2.7.4, statement of Proposition 2.7.6; homework assigned Jan. 31, problems 1, 2, 3.

- 2.8. Completeness. You should know what a Cauchy sequence is and the statement of Theorem 2.8.5; homework assigned Jan. 31, problem 4.

- 2.9. Infinite series. Omit.

- 3.1–3.3. Compactness. The following material is from lectures (we didn't follow the text).

  1. Definition of a bounded set.
  2. Statement of Nested Sets Property as done in lecture.
  3. Statement of Bolzano-Weierstrass Theorem as done in lecture.
  4. Open cover definition of compactness; homework assigned Feb. 7, problems 1, 2, 3, 4.

- 4.1. Continuity. Definition 4.1.1, the rephrasing of Definition 4.1.2, and Definition 4.1.3 (especially the second sentence); statement from lecture of "Easy Theorem 4.1.4" and proofs that (i) $\Rightarrow$ (ii), (iii) $\Rightarrow$ (i), and (i) $\Rightarrow$ (iii) for functions from $\mathbb{R}^n$ to $\mathbb{R}^m$ (the last is homework assigned Feb. 7, problem 5); proof that $f(x_1, x_2) = x_1$ is continuous, and use of this result to do problem 3 on p. 108.

- 4.2. Images of compact sets under continuous maps. Proof of Theorem 4.2.1 using subsequences. Homework assigned Feb. 14, problems 1, 2.

- 4.4. Boundedness of continuous functions on compact sets. Statement of Theorem 4.4.1. Proof from lecture that if K is compact and x is a point, then there is a point in K that is closest to x. Homework assigned Feb. 14, problem 4.

- 4.3 Operations on continuous mappings. Proof of Theorem 4.3.1 for $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^p$, statements of Proposition 4.3.2 and Corollary 4.3.3.

- 3.4, 4.2, 4.5. Path-connected sets. Definition, proof that $D(0, \epsilon)$ is path connected, proof of Theorem 4.2.1 for path-connected (not connected) sets. Homework assigned Feb. 14, problems 3, 5.

- 4.6. Uniform continuity: Omit.