MA 426-001/591M-001 Homework

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Assigned April 14, 2003, due April 28, 2003

1. Let $A \subset \mathbb{R}^n$ be bounded, let $f : A \to \mathbb{R}$ be integrable, and let c be a constant. Prove: cf is integrable, and $\int_A cf = c \int_A f$. (This is one of our basic properties of the integral.)

2. Let $A \subset \mathbb{R}^3$ be $\{(x, y, z) : x^2 + y^2 \leq 1 \text{ and } z = 0\}$. Let $f : A \to \mathbb{R}$ be a bounded function. Using the definition of the Riemann integral, show that $\int_A f = 0$.

3. Let $A \subset \mathbb{R}^n$ be bounded, let $f : A \to \mathbb{R}$ be bounded, and assume $f(x) = 0$ except on a set of volume 0. Using the definition of the Riemann integral, show that $\int_A f = 0$.

4. Let $A \subset \mathbb{R}^n$ be bounded, let $f, g : A \to \mathbb{R}$ be integrable, and assume $f(x) = g(x)$ except on a set of volume 0. Using the previous problem, prove: $\int_A f = \int_A g$. Suggestion: This is easy. Look at $\int_A f - g$.

5. Suppose $f : [a, b] \to \mathbb{R}^n$, $n \geq 2$, is C^1.

   (a) Prove: There exists a number $M > 0$ such that for all $t \in [a, b]$, $\|f'(t)\| \leq M$.

   (b) Prove: $f([a, b])$ has volume 0 in $\mathbb{R}^n$. Suggestion: Divide $[a, b]$ into N equal subintervals I_i. Choose $t_i \in I_i$. Find a rectangle around $f(t_i)$ that contains $f(I_i)$.