MA 426-003/591M-003 Homework

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Assigned February 14, 2003, due February 21, 2003

1. Let $K \subset \mathbb{R}^n$ be compact. Let $f : K \to \mathbb{R}^m$ be continuous and one-to-one. Let $g : f(K) \to \mathbb{R}^n$ be the inverse function of f. Show that g is continuous. (Suggestion: Theorem 4.1.4 (iv).)

2. Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be continuous. Let $K \subset \mathbb{R}^n$ be compact. Prove that $f(K)$ is compact by showing that every open cover of $f(K)$ has a finite subcover. Hint: If $U \subset \mathbb{R}^m$ is open, so is $f^{-1}(U)$.

3. Sec. 4.2, problem 1. For "connected" substitute "path connected." Where you say "yes," cite a theorem; where you say "no," give an example.

4. Sec. 4.4, problem 3.

5. Sec. 4.5, problem 2. For "connected" substitute "path connected." Omit the generalization. Note: $\mathbb{R}^n \times \mathbb{R}^m = \mathbb{R}^{n+m}$.