MA 426-001/591M-001 Homework

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Assigned January 14, 2003, Due January 17, 2003

1. Prove that the standard inner product on $\mathbb{R}^n$ satisfies properties (I3) and (I4) of inner products.

2. Suppose x and y are vectors in $\mathbb{R}^n$, $x \neq 0$, $y \neq 0$, and $< x, y >= \|x\|\|y\|$. Prove that there is a number $\alpha > 0$ such that $x = \alpha y$. Suggestion: Look carefully at the proof of the Cauchy-Schwartz inequality.

3. On $\mathbb{R}^n$, define $\|x\|$ to be $\max_{1 \leq i \leq n} |x_i|$. Prove that $\| \cdot \|$ is a norm. In other words, show that $\| \cdot \|$ satisfies properties (N1)–(N4). ($\| \cdot \|$ is called the "sup norm.")

4. Let S be any set. For any x and y in S, define $d(x, y)$ to be 0 if $x = y$ and 1 if $x \neq y$. Prove that d is a metric. In other words, show that d satisfies properties (D1)–(D4). (d is called the "discrete metric."