MA 425-003 Test 1

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1. Group 1.

   Do *four* problem from Group 1. Give careful proofs from first principles.

   (a) Let S be a nonempty subset of $\mathbb{R}$ that is bounded above. Let $u = \sup S$. Let $a > 0$. Let $aS = \{as : s \in S\}$. Show that $\sup aS = au$.

   (b) Let $x_n = 2 + \frac{(-1)^n}{n^2}$. Prove that $x_n \to 2$.

   (c) Let (x_n) be a sequence such that $x_n \to x$. Suppose $x > a$. Prove that there is a number N such that $x_n > a$ for all $n > N$.

   (d) Let (x_n) be a sequence such that (1) $x_n > a$ for every n and (2) $x_n \to x$. Prove that $x \geq a$.

   (e) Prove: If (x_n) is a bounded increasing sequence and $u = \sup\{x_n : n \in \mathbb{N}\}$, then $x_n \to u$.

2. Group 2.

   Answer *all* questions in Group 2.

   (a) Let (x_n) be a sequence such that (1) $x_n > a$ for every n and (2) $x_n \to x$. Is $x > a$?

      i. Yes.

      ii. No.

      iii. Maybe.

(b) Let (x_n) be a monotone increasing sequence. Does (x_n) converge?

i. Yes.

ii. No.

iii. Maybe.

(c) Let (x_n) be a bounded sequence. Does (x_n) converge?

i. Yes.

ii. No.

iii. Maybe.

(d) Suppose (x_n) converges and (x_{n_k}) is a subsequence of (x_n). Does (x_{n_k}) converge?

i. Yes.

ii. No.

iii. Maybe.

(e) Let (x_n) be a sequence such that (1) $1 < x_n < 2$ for every n and (2) $x_{n+1} < x_n$ for every n. Does $x_n \to 1$?

i. Yes.

ii. No.

iii. Maybe.