MA 341-001 Test 3

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Use your own paper to work the problems. On all problems, you must **show your work** to receive credit.

Don't do more than is asked for!

When you finish, fold this paper lengthwise together with your work, so that this writing is on the outside. Write your name, row number, and seat number above, and turn in.

1. Consider the differential equation

$$\mathbf{x}'(t) = \begin{bmatrix} 0 & 2 \\ -8 & 0 \end{bmatrix} \mathbf{x}(t).$$

   Let

$$\mathbf{x}_1(t) = \begin{bmatrix} \cos 4t \\ -2\sin 4t \end{bmatrix} \quad \text{and} \quad \mathbf{x}_2(t) = \begin{bmatrix} \sin 4t \\ 2\cos 4t \end{bmatrix}.$$

   (a) Check that $\mathbf{x}_1(t)$ is a solution. (Just substitute and check that it works.)

   (b) $\mathbf{x}_2(t)$ is also a solution. (You don't need to check this.) Show that $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ are linearly independent.

   (c) The general solution is

$$\mathbf{x}(t) = c_1 \begin{bmatrix} \cos 4t \\ -2\sin 4t \end{bmatrix} + c_2 \begin{bmatrix} \sin 4t \\ 2\cos 4t \end{bmatrix}.$$

   Give the solution that satisfies the initial condition

$$\mathbf{x}(0) = \begin{bmatrix} -3 \\ 2 \end{bmatrix}.$$

2. Find the eigenvalues of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ -4 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}.$$

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3. The matrix
$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{bmatrix}$$

has the eigenvalues $r = -1, -2, 3$. Eigenvectors for the eigenvalues -1 and -2 are
$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}.$$

respectively.

  (a) Find an eigenvector for the eigenvalue 3.

  (b) Give the general solution of $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$.

4. The matrix
$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

has the eigenvalues $r = 2 \pm i$. Find the real general solution of $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$.

5. Let
$$\mathbf{A} = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \quad \text{and} \quad \mathbf{f}(t) = \begin{bmatrix} e^t \\ e^t \end{bmatrix}.$$

A fundamental matrix for $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$ is
$$\mathbf{X}(t) = \begin{bmatrix} 3e^t & e^{-t} \\ e^t & e^{-t} \end{bmatrix}.$$

Use the variation of parameters method to find the general solution of
$$\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{f}(t).$$

Recall that the variation of parameters formula for the general solution is
$$\mathbf{x}(t) = \mathbf{X}(t)\mathbf{c} + \mathbf{X}(t) \int \mathbf{X}^{-1}(t)\mathbf{f}(t) \, dt.$$

Also recall that the inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$