MA 341-007 Test 1 Review Questions

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Corrected February 8, 2012

Warning: Not all topics are covered!

1. Translate into a differential equation: Sec. 1.1 problems 15 and 16.
2. Checking whether something is a solution of a differential equation: Sec. 1.2 problems 4 and 12.
3. Existence-uniqueness theorem: Sec. 1.2 problem 28. Also: What if the initial condition is $y(1) = 2$?
4. Direction fields: Sec. 1.3 problem 17 (problem 16 in the 5th edition). Use the method of isoclines to sketch the direction field in the region $x > 0$. Then try to sketch the solutions with $y(1) = 1$ and $y(1) = 5$. For these two solutions, as $x \to \infty$, what do you think y approaches?
5. Euler's method: Sec. 1.4 problem 6 (problem 5 in the 5th edition). Just do the points $x = 1.2, 1.4$. Don't round.
6. Separable equations: Sec. 2.2 problem 26.
7. Linear equations: Sec. 2.3 problems 14, 15.
8. Exact equations: Sec. 2.4 problem 12.
9. Mixing: Sec. 3.2 problem 2.
10. Mechanics: Sec. 3.4 problem 6. First draw your coordinate axis!

Answers:

1. 16: $\frac{dA}{dt} = kA^2$.
2. 4 yes, 12 yes. Just calculate $\frac{dy}{dx}$ and plug into the differential equation.
3. No, yes. Look at $\frac{\partial f}{\partial y}$ at the two points.
4. The line $x + 2y = -\frac{1}{2}$ is a solution. Solutions above the line approach the line as $x \to -\infty$ and have $y \to \infty$ as x increases. Solutions below the line approach the line as $x \to -\infty$ and have $y \to -\infty$ as x increases.
5. Approximation from Euler's method: At $x = 1.2$, $y = 1.400$; at $x = 1.4$, $y = 1.960$.
6. $y = \left(1 - \frac{1}{2}\ln(1+x)\right)^2$
7. 14: $y = x^{-3}\sin x - x^{-2}\cos x - \frac{3}{5}x^2 + cx^{-3}$
8. $e^x \sin y - x^3 + y^{\frac{1}{3}} = C$
9. $x = 2.5 - 2.0e^{-.12t}$; t $= \frac{25}{3}\ln 2 = 5.776$.
10. If x increases as you go up, $v = -4.9 + 24.9e^{-2t}$.

    If the initial position of the object is $x = 0$, $x = -4.9t - 12.45e^{-2t} + 12.45$.

    Then $x = -100$ when (after simplifying) $22.95 = t + 2.54e^{-2t}$.

    Solution according to Maple: $t = 22.95$. (Not surprising.)