

Test 2 Answers

$$\textcircled{1} \quad \text{a) } y'' + 4y = te^{-2t}$$

Homogeneous equation:

$$y'' + 4y = 0$$

$$r^2 + 4 = 0$$

$$r = \pm 2i$$

$$y_h = C_1 \cos 2t + C_2 \sin 2t$$

$$y_p = (At + B)e^{-2t}$$

$$\text{b) } y'' + 4y = 6\sin 2t$$

Homogeneous equation:

Same

$$y_p = t(A\cos 2t + B\sin 2t)$$

\textcircled{2}

Homogeneous equation:

$$y'' + 3y' + 2y = 0$$

$$r^2 + 3r + 2 = 0$$

$$(r+1)(r+2) = 0$$

$$r = -1, -2$$

$$y_h = C_1 e^{-t} + C_2 e^{-2t}$$

$$y_p = At^2 + Bt + C$$

$$y'_p = 2At + B$$

$$y''_p = 2A$$

$$2A + 3(2At + B) + 2(At^2 + Bt + C) = 8t^2 - 30$$

$$t^2: \quad 2A = 8$$

$$t: \quad 6A + 2B = 0$$

$$1: \quad 2A + 3B + 2C = -30$$

$$A = 4, B = -12, C = -1$$

General Solution: $y = y_h + y_p = c_1 e^{-t} + c_2 e^{-2t} + 4t^2 - 12t - 1$

③ $y'' - 2y' + y = t^3 e^t$

Homogeneous equation: $y'' - 2y' + y = 0$

$$r^2 - 2r + 1 = 0$$

$$(r-1)^2 = 0$$

$$r=1, 1$$

$$y_h = c_1 \underline{e^t} + c_2 \underline{te^t}$$

$$\begin{matrix} " \\ y_1 \\ " \\ y_2 \end{matrix}$$

$$y_1 y'_2 - y'_1 y_2 = e^t (te^t + e^t) - e^t \cdot te^t = e^{2t}$$

$$v_1 = \int \frac{-t^3 e^t \cdot te^t}{1 \cdot e^{2t}} dt = \int -t^2 dt = t^{-1} \cancel{\text{}}$$

$$v_2 = \int \frac{t^3 e^t \cdot e^t}{1 \cdot e^{2t}} dt = \int t^3 dt = -\frac{1}{2} t^2 \cancel{\text{}}$$

$$y_p = v_1 y_1 + v_2 y_2 = t^{-1} e^t \bullet -\frac{1}{2} t^2 \cdot te^t = \underbrace{\frac{1}{2} t^{-1} e^t}_{\text{in}}$$

④ $y'' + 6y' + 9y = t^3 e^{-5t} \quad y(0) = 2 \quad y'(0) = -3$

Let $\mathcal{L}\{y(t)\} = Y(s)$. Transform both sides:

$$(s^2 Y - s y(0) - y'(0)) + 6(sY - y(0)) + 9Y = \frac{3!}{(s+5)^4}$$

$$(s^2Y - 2s + 3) + 6sY - 12 + 9Y = \frac{6}{(s+5)^4}$$

$$(s^2 + 6s + 9)Y = \frac{6}{(s+5)^4} + 2s + 9$$

$$(s+3)^2 Y = \frac{6 + (2s+9)(s+5)^4}{(s+5)^4}$$

$$Y = \frac{6 + (2s+9)(s+5)^4}{(s+5)^4(s+3)^2}$$

$$\textcircled{5} \quad a) \quad \frac{10-2s}{s^2+2s+10} = \frac{10-2s}{(s+1)^2+9} = A \frac{s+1}{(s+1)^2+9} + B \frac{3}{(s+1)^2+9}$$

$$= \frac{A(s+1) + 3B}{(s+1)^2 + 9} \Rightarrow \begin{array}{l} s: A = -2 \\ 1: A + 3B = 10 \rightarrow B = 4 \end{array}$$

$$\Rightarrow \mathcal{L}^{-1} \left\{ \frac{10-2s}{s^2+2s+10} \right\} = -2e^{-t} \cos 3t + 4e^{-t} \sin 3t$$

$$\begin{aligned} b) \quad \frac{s^2-4}{s^2(s-1)} &= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} = \frac{As(s-1) + B(s-1) + Cs^2}{s^2(s-1)} \\ &= \frac{(A+C)s^2 + (B-A)s - B}{s^2(s-1)} \Rightarrow \begin{array}{l} s^2: A+C = 1 \\ s: B-A = 0 \\ 1: -B = -4 \end{array} \end{aligned}$$

$$\Rightarrow B=4, A=1, C=-3 \Rightarrow$$

$$\mathcal{L}^{-1} \left\{ \frac{s^2-4}{s^2(s-1)} \right\} = \mathcal{L}^{-1} \left\{ \frac{4}{s} + \frac{1}{s^2} - \frac{3}{s-1} \right\} = 4 + 4t - 3e^t$$

$$c) \mathcal{L}^{-1} \left\{ \frac{1}{(s+4)^3} \right\} = \mathcal{L}^{-1} \left\{ \frac{1}{2!} \frac{2!}{(s+4)^3} \right\} = \frac{1}{2} e^{-4t} t^2$$

$$\mathcal{L}^{-1} \left\{ e^{-6s} \cdot \frac{1}{(s+4)^3} \right\} = \frac{1}{2} e^{-4(t-6)} (t-6)^2 h(t-6)$$

Points

① 14

② 20

③ 20

④ 12

⑤ a) 12

b) 12

c) 10