

## Test 1 Answers

$$\textcircled{1} \text{ a) } 4 \ln y - xy^2 = C$$

$$4 \cdot \frac{1}{y} \frac{dy}{dx} - (1 \cdot y^2 + x \cdot 2y \frac{dy}{dx}) = 0$$

$$\left(\frac{4}{y} - 2xy\right) \frac{dy}{dx} = y^2$$

$$\frac{dy}{dx} = \frac{y^2}{\frac{4}{y} - 2xy} = \frac{y^3}{4 - 2xy^2}$$

$$\text{Substitute into d.e.: } \frac{y^3}{4 - 2xy^2} \stackrel{?}{=} \frac{y^3}{4 - 2xy^2} \checkmark$$

$$\text{b) } 4 \ln 1 - 2 \cdot 1^2 = C$$

$$\textcircled{-2 = C}$$

c) We have  $\frac{dy}{dx} = f(x, y) = \frac{y^3}{4 - 2xy^2}$ . At  $(x_0, y_0) = (2, 1)$ , this expression is not defined; we are dividing by 0. Therefore the Existence-Uniqueness Theorem does not guarantee a unique solution.

$$\textcircled{2} \quad y_{n+1} = y_n + hf(x_n, y_n) \quad h = 0.1 \quad f(x, y) = 2x + y^2$$

| n | $x_n$ | $y_n$ | $f(x_n, y_n)$            | $hf(x_n, y_n)$ |
|---|-------|-------|--------------------------|----------------|
| 0 | 1     | 0     | 2                        | .2             |
| 1 | 1.1   | .2    | $2(1.1) + (.2)^2 = 2.24$ | .224           |
| 2 | 1.2   | .424  |                          |                |

$$y(1.1) \approx .2, \quad y(1.2) \approx .424$$

$$\textcircled{3} \int (y+1)^{-\frac{1}{2}} dy = \int \frac{dx}{2x+1}$$

$$2(y+1)^{\frac{1}{2}} = \frac{1}{2} \ln(2x+1) + C$$

$$(y+1)^{\frac{1}{2}} = \frac{1}{4} \ln(2x+1) + K$$

$$y+1 = \left(\frac{1}{4} \ln(2x+1) + K\right)^2$$

$$y = -1 + \left(\frac{1}{4} \ln(2x+1) + K\right)^2$$

$$\textcircled{4} \frac{dy}{dx} + \frac{4}{x}y = \frac{1}{x^2}$$

Multiply by  $e^{\int \frac{4}{x} dx} = e^{4 \ln x} = x^4$

$$x^4 \frac{dy}{dx} + 4x^3 y = x^2$$

$$\frac{d}{dx}(x^4 y) = x^2$$

$$x^4 y = \frac{1}{3} x^3 + C$$

$$y = \frac{1}{3x} + \frac{C}{x^4}$$

$$\textcircled{5} \underbrace{(2y+1) \cos 2x}_{M(x,y)} dx + \underbrace{(y^2 + \sin 2x)}_{N(x,y)} dy = 0$$

$$\text{a) } M_y = 2 \cos 2x \quad N_x = 2 \cos 2x$$

$M_y = N_x$  so exact.

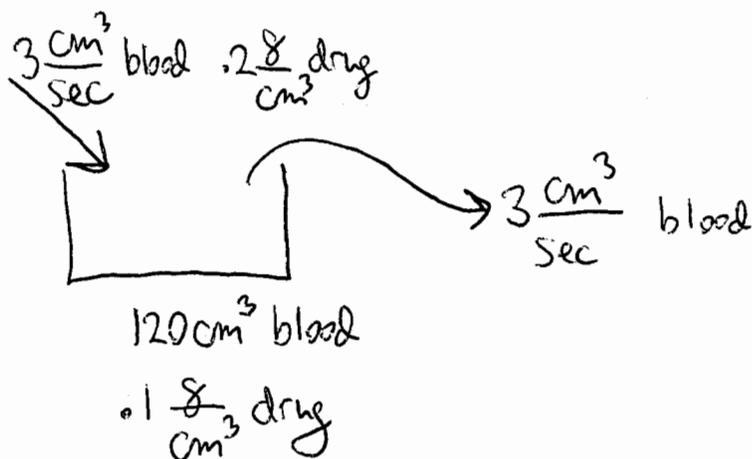
$$\begin{aligned}
 \text{b) } F(x,y) &= \int (2y+1) \cos 2x \, dx \\
 &= \int 2y \cos 2x + \cos 2x \, dx \\
 &= y \sin 2x + \frac{1}{2} \sin 2x + g(y)
 \end{aligned}$$

$$\frac{\partial F}{\partial y} = \sin 2x + g'(y) = y^2 + \sin 2x$$

$$g(y) = \frac{1}{3} y^3$$

$$\underline{y \sin 2x + \frac{1}{2} \sin 2x + \frac{1}{3} y^3 = C}$$

(6)



$x$  = grams of drug     $t$  = Time in seconds

$$\text{a) } \frac{dx}{dt} = 3 \frac{\text{cm}^3}{\text{sec}} \cdot 0.2 \frac{\text{g}}{\text{cm}^3} - 3 \frac{\text{cm}^3}{\text{sec}} \cdot \frac{x}{120} \frac{\text{g}}{\text{cm}^3}$$

$\frac{\text{g}}{\text{sec}}$

$$\text{So } \underline{\frac{dx}{dt} = 0.6 - 0.075x} \quad \text{or} \quad \underline{\frac{dx}{dt} = \frac{3}{5} - \frac{3}{40}x}$$

$$\text{b) } x(0) = 0.1 \frac{\text{g}}{\text{cm}^3} \cdot 120 \text{ cm}^3 = 12$$

Points

① a) 8

b) 4

c) 6

② 16

③ 16

④ 16

⑤ a) 6

b) 16

⑥ a) 8

b) 4