(c) $y(t) = \sin 8t \sin t$

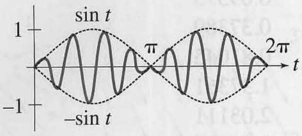

Figure B.28

7. $y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} + \dfrac{F_0 \sin(\gamma t + \theta)}{\sqrt{(k - m\gamma^2)^2 + b^2\gamma^2}}$,
where $r_1, r_2 = -(b/2m) \pm (1/2m)\sqrt{b^2 - 4mk}$ and $\tan\theta = (k - m\gamma^2)/(b\gamma)$ as in equation (7)

9. $y_p(t) = (0.08)\cos 2t + (0.06)\sin 2t$
$= (0.1)\sin(2t + \theta)$, where
$\theta = \arctan(4/3) \approx 0.927$

11. $y(t) = -(18/85)e^{-2t}\cos 6t - (22/255)e^{-2t}\sin 6t$
$+ (2/\sqrt{85})\sin(2t + \theta)$, where
$\theta = \arctan(9/2) \approx 1.352$;
res. freq. $= 2\sqrt{2}/\pi$ cycles/sec

13. $y_p(t) = (3/185)(8\sin 4t - 11\cos 4t)$

15. $\text{Amp} = \sqrt{\left(-\dfrac{11}{986}\right)^2 + \left(-\dfrac{3}{1972}\right)^2} \approx 0.01(\text{m})$,
freq. $= 2/\pi$

Review Problems, page 233

1. $c_1 e^{-9t} + c_2 e^t$

3. $c_1 e^{t/2}\cos(3t/2) + c_2 e^{t/2}\sin(3t/2)$

5. $c_1 e^{3t/2} + c_2 e^{t/3}$

7. $c_1 e^{-t/3}\cos(t/6) + c_2 e^{-t/3}\sin(t/6)$

9. $c_1 e^{7t/4} + c_2 t e^{7t/4}$

11. $t^{1/2}\{c_1\cos[(\sqrt{19}/2)\ln t] + c_2\sin[(\sqrt{19}/2)\ln t]\}$

13. $c_1\cos 4t + c_2\sin 4t + (1/17)te^t - (2/289)e^t$

15. $c_1 e^{-2t} + c_2 e^{-t} + c_3 e^{-t/3}$

17. $c_1 e^t + c_2 e^{-t/2}\cos(\sqrt{43}t/2) + c_3 e^{-t/2}\sin(\sqrt{43}t/2)$

19. $c_1 e^{-3t} + c_2 e^{t/2} + c_3 t e^{t/2}$

21. $c_1 e^{3t/2}\cos(\sqrt{19}t/2) + c_2 e^{3t/2}\sin(\sqrt{19}t/2) - e^t/5$
$+ t^2 + 6t/7 + 4/49$

23. $c_1\cos 4\theta + c_2\sin 4\theta$
$-(1/16)(\cos 4\theta)\ln|\sec 4\theta + \tan 4\theta|$

25. $c_1 e^{3t/2} + c_2 t e^{3t/2} + e^{3t}/9 + e^{5t}/49$

27. $c_1 x + c_2 x^{-2} - 2x^{-2}\ln x + x\ln x$

29. $e^{-2t}\cos(\sqrt{3}t)$

31. $2e^t\cos 3t - (7/3)e^t\sin 3t - \sin 3t$

33. $-e^{-t} - 3e^{5t} + e^{8t}$

35. $\cos\theta + 2\sin\theta + \theta\sin\theta + (\cos\theta)\ln|\cos\theta|$

37. (a), (c), (e), and (f) have all solutions bounded as $t \to +\infty$

39. $y_p(t) = (1/4)\sin 8t$; $\sqrt{62}/2\pi$

CHAPTER 5

Exercises 5.2, page 250

1. **(a)** $-t^3 + 3t^2 + 8$ **(b)** $-2t^3 + 3t^2 + 6t + 16$
(c) $2t^3 + 3t^2 - 16$ **(d)** $-2t^3 + 3t^2 + 6t + 16$
(e) $-2t^3 + 3t^2 + 6t + 16$

3. $x = c_1 + c_2 e^{-2t}$; $y = c_2 e^{-2t}$

5. $x \equiv -5$; $y \equiv 1$

7. $u = c_1 - (1/2)c_2 e^{-t} + (1/2)e^t + (5/3)t$;
$v = c_1 + c_2 e^{-t} + (5/3)t$

9. $x = c_1 e^t + (1/4)\cos t - (1/4)\sin t$;
$y = -3c_1 e^t - (3/4)\cos t - (1/4)\sin t$

11. $u = c_1\cos 2t + c_2\sin 2t + c_3 e^{\sqrt{3}t} + c_4 e^{-\sqrt{3}t}$
$- (3/10)e^t$;
$v = c_1\cos 2t + c_2\sin 2t - (2/5)c_3 e^{\sqrt{3}t}$
$- (2/5)c_4 e^{-\sqrt{3}t} + (1/5)e^t$

13. $x = 2c_2 e^t\cos 2t - 2c_1 e^t\sin 2t$;
$y = c_1 e^t\cos 2t + c_2 e^t\sin 2t$

15. $w = -(2/3)c_1 e^{2t} + c_2 e^{7t} + t + 1$;
$z = c_1 e^{2t} + c_2 e^{7t} - 5t - 2$

17. $x = c_1\cos t + c_2\sin t - 4c_3\cos(\sqrt{6}t)$
$- 4c_4\sin(\sqrt{6}t)$;
$y = c_1\cos t + c_2\sin t + c_3\cos(\sqrt{6}t) + c_4\sin(\sqrt{6}t)$

19. $x = 2e^{3t} - e^{2t}$;
$y = -2e^{3t} + 2e^{2t}$

21. $x = e^t + e^{-t} + \cos t + \sin t$;
$y = e^t + e^{-t} - \cos t - \sin t$

23. Infinitely many solutions satisfying
$x + y = e^t + e^{-2t}$

25. $x(t) = -c_1 e^t - 2c_2 e^{2t} - c_3 e^{3t}$,
$y(t) = c_1 e^t + c_2 e^{2t} + c_3 e^{3t}$,
$z(t) = 2c_1 e^t + 4c_2 e^{2t} + 4c_3 e^{3t}$

27. $x(t) = c_1 e^{8t} + c_2 e^{4t} + c_3$,
$y(t) = \dfrac{1}{2}(c_1 e^{8t} - c_2 e^{4t} + c_3)$,
$z(t) = -c_1 e^{8t} + c_3$

29. $-3 \le \lambda \le -1$

31. $x(t) = -\left(10 + \dfrac{20}{\sqrt{7}}\right)e^{r_1 t} - \left(10 - \dfrac{20}{\sqrt{7}}\right)e^{r_2 t}$
$+ 20$ kg,
$y(t) = \dfrac{30}{\sqrt{7}}e^{r_1 t} - \dfrac{30}{\sqrt{7}}e^{r_2 t} + 20$ kg, where
$r_1 = \dfrac{-5 - \sqrt{7}}{100} \approx -0.0765$,
$r_2 = \dfrac{-5 + \sqrt{7}}{100} \approx -0.0235$

33. $x = \left[\frac{20 - 10\sqrt{5}}{\sqrt{5}}\right] e^{(-3+\sqrt{5})t/100}$
$- \left[\frac{20 + 10\sqrt{5}}{\sqrt{5}}\right] e^{(-3-\sqrt{5})t/100} + 20\ ;$
$y = -\left(\frac{10}{\sqrt{5}}\right) e^{(-3+\sqrt{5})t/100} + \left(\frac{10}{\sqrt{5}}\right) e^{(-3-\sqrt{5})t/100}$
$+ 20$

35. 90.4ºF **37.** $460/11 \approx 41.8$ºF

Exercises 5.3, page 260

1. $x_1' = x_2\ ,\quad x_2' = 3x_1 - tx_2 + t^2\ ;\quad x_1(0) = 3\ ,$
$x_2(0) = -6$

3. $x_1' = x_2\ ,\quad x_2' = x_3\ ,\quad x_3' = x_4\ ,$
$x_4' = x_4 - 7x_1 + \cos t\ ;\quad x_1(0) = x_2(0) = 1\ ,$
$x_3(0) = 0\ ,\quad x_4(0) = 2$

5. $x_1' = x_2\ ,\quad x_2' = x_2 - x_3 + 2t\ ,$
$x_3' = x_4\ ,\quad x_4' = x_1 - x_3 - 1\ ;$
$x_1(3) = 5\ ,\quad x_2(3) = 2\ ,\quad x_3(3) = 1\ ,\quad x_4(3) = -1$

7. $x_1' = x_2\ ,\quad x_2' = x_3\ ,\quad x_3' = x_4 + t\ ,\quad x_4' = x_5\ ,$
$x_5' = (2x_4 - 2x_3 + 1)/5\ ;\quad x_1(0) = x_2(0) = x_3(0) = 4\ ,$
$x_4(0) = x_5(0) = 1$

9. $t_{n+1} = t_n + h,\quad n = 0, 1, 2, \ldots;$
$x_{i,n+1} = x_{i,n} + \frac{h}{2}\left[f_i(t_n, x_{1,n}, \ldots, x_{m,n}) + \right.$
$f_i(t_n + h, x_{1,n} + hf_1(t_n, x_{1,n}, \ldots, x_{m,n}), \ldots,$
$\left. x_{m,n} + hf_m(t_n, x_{1,n}, \ldots, x_{m,n})\right]$
$i = 1, 2, \ldots, m$

11.

i	t_i	$y(t_i)$
1	0.250	0.750000
2	0.500	0.625000
3	0.750	0.573529
4	1.000	0.563603

13.

i	t_i	$y(t_i)$
1	0.250	0.25000
2	0.500	0.50000
3	0.750	0.75000
4	1.000	1.00000

15. $y(1) \approx x_1(1, 2^{-2}) = 1.69, y'(1) \approx 1.82$

17. $u(1; 2^{-2}) = v(1; 2^{-2}) = 0.36789$

19.

		Part (a)		Part (b)		Part (c)	
i	t_i	$x(t_i)$	$y(t_i)$	$x(t_i)$	$y(t_i)$	$x(t_i)$	$y(t_i)$
1	0.5	1.95247	2.25065	1.48118	2.42311	0.91390	2.79704
2	1.0	3.34588	1.83601	2.66294	1.45358	1.63657	1.13415
3	1.5	4.53662	3.36527	5.19629	2.40348	4.49334	1.07811
4	2.0	2.47788	4.32906	3.10706	4.64923	5.96115	5.47788
5	2.5	1.96093	2.71900	1.92574	3.32426	1.51830	5.93110
6	3.0	2.86412	1.96166	2.34143	2.05910	0.95601	2.18079
7	3.5	4.28449	2.77457	3.90106	2.18977	2.06006	0.98131
8	4.0	3.00965	4.11886	3.83241	3.89043	5.62642	1.38072
9	4.5	2.18643	3.14344	2.32171	3.79362	5.10594	5.10462
10	5.0	2.63187	2.25824	2.21926	2.49307	1.74187	5.02491

21.

i	t_i	$x_1(t_i) \approx H(t_i)$
1	0.5	0.09573
2	1.0	0.37389
3	1.5	0.81045
4	2.0	1.37361
5	2.5	2.03111
6	3.0	2.75497
7	3.5	3.52322
8	4.0	4.31970
9	4.5	5.13307
10	5.0	5.95554

23. Yes, yes

25. $y(1) \approx x_1(1; 2^{-3}) = 1.25958$

27. $y(0.1) \approx 0.00647, \ldots, y(2.0) \approx 1.60009$

29. **(a)** $P_1(10) \approx 0.567, P_2(10) \approx 0.463, P_3(10) \approx 0.463$
(b) $P_1(10) \approx 0.463, P_2(10) \approx 0.567, P_3(10) \approx 0.463$
(c) $P_1(10) \approx 0.463, P_2(10) \approx 0.463, P_3(10) \approx 0.567$
All populations approach 0.5.

Exercises 5.4, page 272

1. $x = y^3\ ,\quad y > 0$

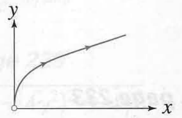

Figure B.29

3. $\left(1/\sqrt{2}, 1/\sqrt{2}\right),\quad \left(-1/\sqrt{2}, -1/\sqrt{2}\right)$ **5.** $(0, 0)$

7. $e^x + ye^{-y} = c$ **9.** $e^x + xy - y^2 = c$

11. $y^2 - x^2 = c$ **13.** $(x - 1)^2 + (y - 1)^2 = c$

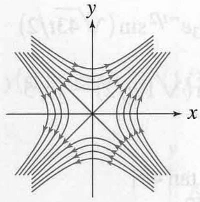

Figure B.30

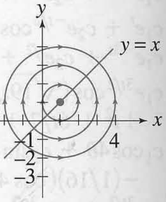

Figure B.31

15. $(-2, 1)$ is a saddle point (unstable).

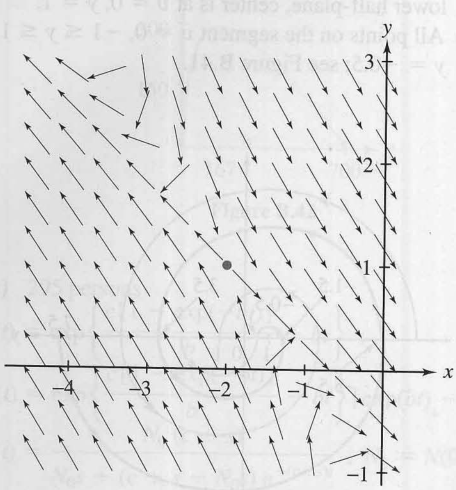

Figure B.32

17. $(0, 0)$ is a center (stable).

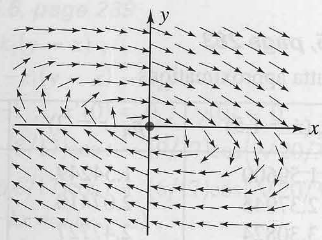

Figure B.33

19. $(0, 0)$ is a saddle point (unstable).

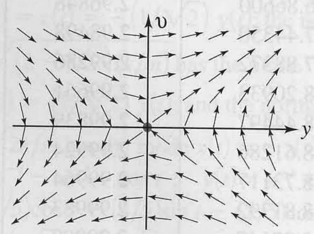

Figure B.34

21. $(0, 0)$ is a center (stable).

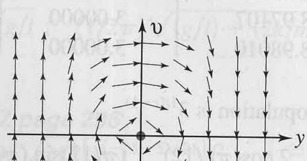

Figure B.35

23. $(0, 0)$ is a center (stable); $(1, 0)$ is a saddle point (unstable).

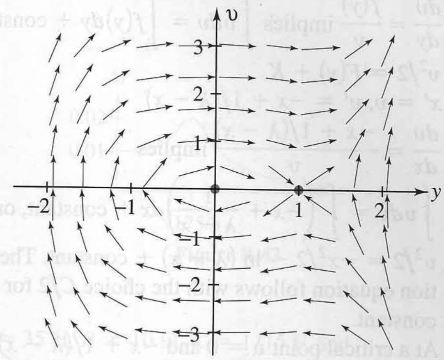

Figure B.36

25. **(a)** Periodic **(b)** Not periodic
(c) Critical point (periodic)

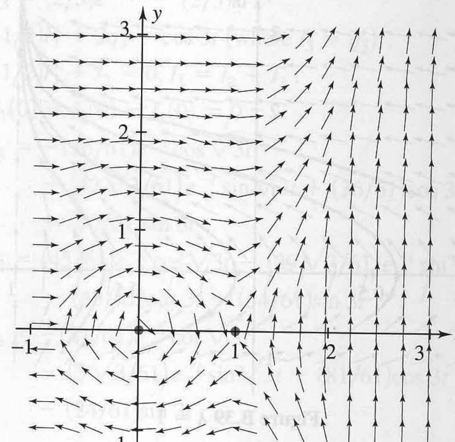

Figure B.37

27. $(x(t), y(t))$ approaches $(0, 0)$.

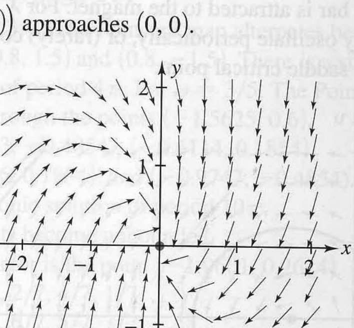

Figure B.38

29. **(a)** Unstable node **(b)** Center (stable)
(c) Stable node **(d)** Unstable spiral
(e) Saddle (unstable) **(f)** Asymptotically stable spiral

31. **(a)** $y' = v\,,\quad v' = f(y)$

(b) $\dfrac{dv}{dy} = \dfrac{f(y)}{v}$ implies $\int v\,dv = \int f(y)dy$ + constant, or $v^2/2 = F(y) + K$

33. **(a)** $x' = v, v' = -x + 1/(\lambda - x)$

(b) $\dfrac{dv}{dx} = \dfrac{-x + 1/(\lambda - x)}{v}$ implies

$$\int v\,dv = \int \left(-x + \frac{1}{\lambda - x}\right)dx + \text{constant, or}$$

$v^2/2 = -x^2/2 - \ln(\lambda - x)$ + constant. The solution equation follows with the choice $C/2$ for the constant.

(c) At a critical point $v = 0$ and $-x + 1/(\lambda - x) = 0$. Solutions for the latter are $x = \dfrac{\lambda \pm \sqrt{\lambda^2 - 4}}{2}$, and are real only for $\lambda \geq 2$.

(d)

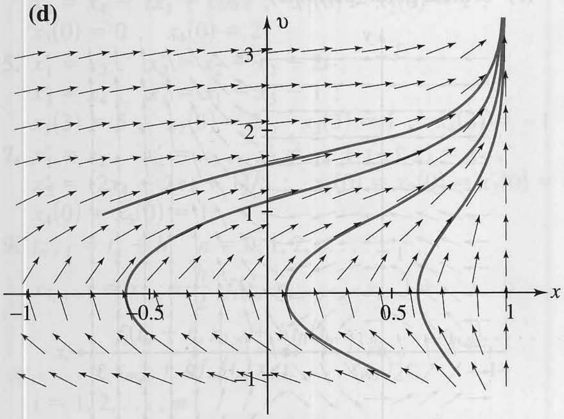

Figure B.39 $\lambda = 1$

(e) See Figures B.39 and B.40. When $\lambda = 3$, one critical point is a center and the other is a saddle. For $\lambda = 1$, the bar is attracted to the magnet. For $\lambda = 3$, the bar may oscillate periodically, or (rarely) come to rest at the saddle critical point.

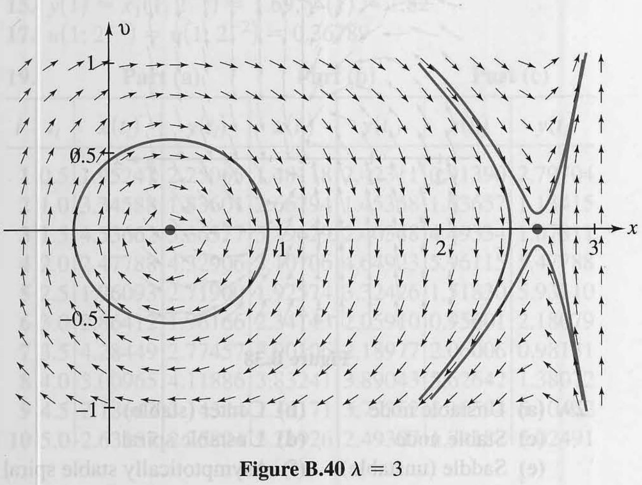

Figure B.40 $\lambda = 3$

35. **(c)** For upper half-plane, center is at $v = 0, y = -1$; for lower half-plane, center is at $v = 0, y = 1$.

(d) All points on the segment $v = 0, -1 \leq y \leq 1$

(e) $y = -0.5$; see Figure B.41.

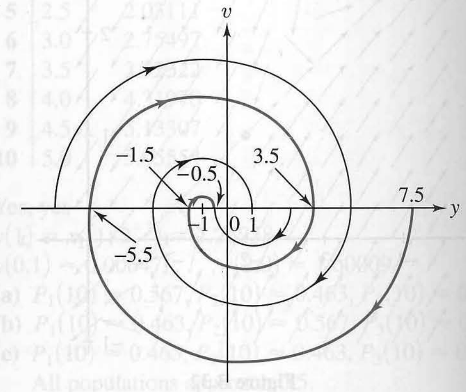

Figure B.41

Exercises 5.5, page 283

1. Runge–Kutta approximations

t_n	$p_n\ (r = 1.5)$	$p_n\ (r = 2)$	$p_n\ (r = 3)$
0.25	1.59600	1.54249	1.43911
0.5	2.37945	2.07410	1.64557
0.75	3.30824	2.47727	1.70801
1.0	4.30243	2.72769	1.72545
1.25	5.27054	2.86458	1.73024
1.5	6.13869	2.93427	1.73156
1.75	6.86600	2.96848	1.73192
2.0	7.44350	2.98498	1.73201
2.25	7.88372	2.99286	1.73204
2.5	8.20933	2.99661	1.73205
2.75	8.44497	2.99839	1.73205
3.0	8.61286	2.99924	1.73205
3.25	8.73117	2.99964	1.73205
3.5	8.81392	2.99983	1.73205
3.75	8.87147	2.99992	1.73205
4.0	8.91136	2.99996	1.73205
4.25	8.93893	2.99998	1.73205
4.5	8.95796	2.99999	1.73205
4.75	8.97107	3.00000	1.73205
5.0	8.98010	3.00000	1.73205

Limiting population is $3^{1/(r-1)}$.

3. $$x(t) = \frac{1}{2} - \frac{2\cos(\pi t/12)}{4 + (\pi/12)^2} - \frac{(\pi/12)\sin(\pi t/12)}{4 + (\pi/12)^2} + \left(\frac{19}{2} + \frac{2}{4 + (\pi/12)^2}\right)e^{-2t}$$

5. (a)

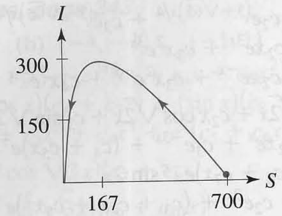

Figure B.42

(b) 295 persons

7. $P(t) = \exp\left\{\dfrac{c[1-\exp(-bt)]}{b} - bt\right\}$,

$Q(t) = \exp\left\{\dfrac{c[1-\exp(-bt)]}{b} - bt\right\}[\exp(bt) - 1]$

9. $N(t) = \dfrac{N_0\,(c+s)}{N_0 s + (c+s-N_0 s)\,e^{-(c+s)t}}$, $N_0 := N(0)$

11. Roughly, 2

Exercises 5.6, page 289

1. $m_1 x'' = k_1(y - x)$,
$m_2 y'' = -k_1(y-x) - k_2 y$;
$x(0) = -1$, $x'(0) = 0$, $y(0) = 0$, $y'(0) = 0$
$x(t) = -(8/17)\cos t - (9/17)\cos\left(\sqrt{20/3}\,t\right)$,
$y(t) = -(6/17)\cos t + (6/17)\cos\left(\sqrt{20/3}\,t\right)$

3. $mx'' = -kx + k(y - x)$,
$my'' = -k(y-x) + k(z-y)$,
$mz'' = -k(z-y) - kz$;
The normal frequency $(1/2\pi)\sqrt{\left(2+\sqrt{2}\right)(k/m)}$ has the mode $x(t) = z(t) = -\left(1/\sqrt{2}\right)y(t)$; the normal frequency $(1/2\pi)\sqrt{\left(2-\sqrt{2}\right)(k/m)}$ has the mode $x(t) = z(t) = \left(1/\sqrt{2}\right)y(t)$; and the normal frequency $(1/2\pi)\sqrt{2k/m}$ has the mode $x(t) = -z(t)$, $y(t) \equiv 0$.

5. $x(t) = -e^{-t} - te^{-t} - \cos t$; $y(t) = e^{-t} + te^{-t} - \cos t$

7. $x(t) = (2/5)\cos t + (4/5)\sin t - (2/5)\cos\sqrt{6}t$
$+ \left(\sqrt{6}/5\right)\sin\sqrt{6}t - \sin 2t$;
$y(t) = (4/5)\cos t + (8/5)\sin t + (1/5)\cos\sqrt{6}t$
$- \left(\sqrt{6}/10\right)\sin\sqrt{6}t - (1/2)\sin 2t$

9. $(1/2\pi)\sqrt{g/l}$; $(1/2\pi)\sqrt{(g/l) + (2k/m)}$

Exercises 5.7, page 296

1. $I(t) = \left(19/\sqrt{21}\right)\left[e^{\left(-25-5\sqrt{21}\right)t/2} - e^{\left(-25+5\sqrt{21}\right)t/2}\right]$

3. $I_p(t) = (4/51)\cos 20t - (1/51)\sin 20t$; resonance frequency is $5/\pi$.

5. $M(\gamma) = 1/\sqrt{(100 - 4\gamma^2)^2 + 100\gamma^2}$

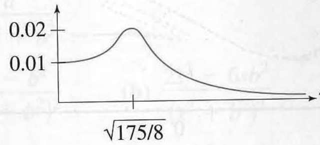

Figure B.43

7. $L = 35$ H, $R = 10\ \Omega$, $C = 1/15$ F, and $E(t) = 50\cos 10t$ V

11. $I_1 = (3/5)e^{-3t/2} - (8/5)e^{-2t/3} + 1$,
$I_2 = (1/5)e^{-3t/2} - (6/5)e^{-2t/3} + 1$,
$I_3 = (2/5)e^{-3t/2} - (2/5)e^{-2t/3}$

13. $(1/2)I_1' + 2q_3 = \cos 3t$ (where $I_3 = q_3'$) ,
$(1/2)I_1' + I_2 = 0$, $I_1 = I_2 + I_3$:
$I_1(0) = I_2(0) = I_3(0) = 0$;
$I_1 = -(36/61)e^{-t}\cos\sqrt{3}t$
$- \left(42\sqrt{3}/61\right)e^{-t}\sin\sqrt{3}t + (36/61)\cos 3t$
$+ (30/61)\sin 3t$
$I_2 = (45/61)e^{-t}\cos\sqrt{3}t - \left(39\sqrt{3}/61\right)e^{-t}\sin\sqrt{3}t$
$- (45/61)\cos 3t + (54/61)\sin 3t$,
$I_3 = -(81/61)e^{-t}\cos\sqrt{3}t$
$- \left(3\sqrt{3}/61\right)e^{-t}\sin\sqrt{3}t + (81/61)\cos 3t$
$- (24/61)\sin 3t$

Exercises 5.8, page 305

1. For $\omega = 3/2$: The Poincaré map alternates between the points $(0.8, 1.5)$ and $(0.8, -1.5)$. There is a subharmonic solution of period 4π. For $\omega = 3/5$: The Poincaré map cycles through the points $(-1.5625, 0.6)$, $(-2.1503, -0.4854)$, $(-0.6114, 0.1854)$, $(-2.5136, 0.1854)$, and $(-0.9747, -0.4854)$. There is a subharmonic solution of period 10π.

3. The points become unbounded.

5. The attractor is the point $(-1.0601, 0.2624)$.

9. **(a)** $\{1/7, 2/7, 4/7, 1/7, \ldots\}$,
$\{3/7, 6/7, 5/7, 3/7, \ldots\}$

(b) $\{1/15, 2/15, 4/15, 8/15, 1/15, \ldots\}$,
$\{1/5, 2/5, 4/5, 3/5, 1/5, \ldots\}$,
$\{1/3, 2/3, 1/3, \ldots\}$,
$\{7/15, 14/15, 13/15, 11/15, 7/15, \ldots\}$

(c) $x_n = 0$ for $n \geq j$

13.

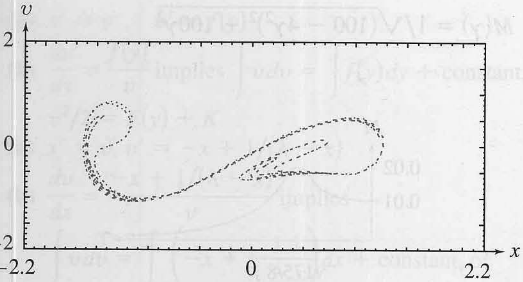

Figure B.44

Review Problems, page 308

1. $x(t) = -(c_1/3)t^3 - (c_2/2)t^2 - (c_3 + 2c_1)t + c_4$, $y(t) = c_1t^2 + c_2t + c_3$

3. $x(t) = c_1 \cos 3t + c_2 \sin 3t + e^t/10$, $y(t) = (3/2)(c_1 + c_2)\cos 3t - (3/2)(c_1 - c_2)\sin 3t - (11/20)e^t - (1/4)e^{-t}$

5. $x(t) = 2 \sin t$, $y(t) = e^t - \cos t + \sin t$, $z(t) = e^t + \cos t + \sin t$

7. $x(t) = -(13.9/4)e^{-t/6} - (4.9/4)e^{-t/2} + 4.8$, $y(t) = -(13.9/2)e^{-t/6} + (4.9/2)e^{-t/2} + 4.8$

9. $x_1' = x_2$, $x_2' = x_3$, $x_3' = (1/3)(5 + e^t x_1 - 2x_2)$

11. $x_1' = x_2$, $x_2' = x_3$, $x_3' = t - x_5 - x_6$, $x_4' = x_5$, $x_5' = x_6$, $x_6' = x_2 - x_3$

13. $x^2 - (y - 1)^2 = c$; critical point $(0, 1)$ is saddle (unstable).

15. Asymptotically stable spiral point

19. $I_1 + I_2 + I_3 = 0$, $q/C = R_2I_2$, $R_2I_2 = R_1I_3 + L\,dI_3/dt$, where q is the charge on the capacitor ($I_1 = dq/dt$); $I_3 = e^{-t}(A \cos t + B \sin t)$, $I_2 = e^{-t}(B \cos t - A \sin t)$, $I_1 = e^{-t}[(A - B)\sin t - (A + B)\cos t]$.

CHAPTER 6

Exercises 6.1, page 325

1. $(-\infty, 0)$ **3.** $(3\pi/2, 5\pi/2)$ **5.** $(0, \infty)$

7. Lin. indep. **9.** Lin. dep.

11. Lin. indep. **13.** Lin. indep.

15. $c_1e^{3x} + c_2e^{-x} + c_3e^{-4x}$ **17.** $c_1x + c_2x^2 + c_3x^3$

19. **(a)** $c_1e^x + c_2e^{-x}\cos 2x + c_3e^{-x}\sin 2x + x^2$ **(b)** $-e^x + e^{-x}\sin 2x + x^2$

21. **(a)** $c_1x + c_2x \ln x + c_3x(\ln x)^2 + \ln x$ **(b)** $3x - x \ln x + x(\ln x)^2 + \ln x$

23. **(a)** $2 \sin x - x$ **(b)** $4x - 6 \sin x$

29. **(b)** Let $f_1(x) = |x - 1|$ and $f_2(x) = x - 1$

33. e^{2x} , $(\sin x - 2 \cos x)/5$, $-(2 \sin x + \cos x)/5$

35. $xy''' - y'' + xy' - y = 0$

Exercises 6.2, page 331

1. $c_1 + c_2e^{2x} + c_3e^{-4x}$ **3.** $c_1e^{-x} + c_2e^{-2x/3} + c_3e^{x/2}$

5. $c_1e^{-x} + c_2e^{-x}\cos 5x + c_3e^{-x}\sin 5x$

7. $c_1e^{-x} + c_2e^{(3+\sqrt{65})x/4} + c_3e^{(3-\sqrt{65})x/4}$

9. $c_1e^{3x} + c_2xe^{3x} + c_3x^2e^{3x}$

11. $c_1e^{-x} + c_2xe^{-x} + c_3x^2e^{-x} + c_4x^3e^{-x}$

13. $c_1 \cos \sqrt{2}x + c_2x \cos \sqrt{2}x + c_3 \sin \sqrt{2}x + c_4x \sin \sqrt{2}x$

15. $c_1e^x + c_2xe^x + c_3e^{-3x} + (c_4 + c_5x)e^{-x}\cos 2x + (c_6 + c_7x)e^{-x}\sin 2x$

17. $c_1e^{-4x} + c_2e^{3x} + (c_3 + c_4x + c_5x^2)e^{-2x} + (c_6 + c_7x)e^{-2x}\cos x + (c_8 + c_9x)e^{-2x}\sin x + c_{10} + c_{11}x + c_{12}x^2 + c_{13}x^3 + c_{14}x^4$

19. $e^x - 2e^{-2x} - 3e^{2x}$ **21.** $e^{2x} - \sqrt{2}e^x \sin \sqrt{2}x$

23. $x(t) = c_1 + c_2t + c_3e^t$, $y(t) = c_1 - c_2 + c_2t$

27. $c_1e^{1.120x} + c_2e^{0.296x} + c_3e^{-0.520x} + c_4e^{-2.896x}$

29. $c_1e^{-0.5x}\cos(0.866x) + c_2\,e^{-0.5x}\sin(0.866x) + c_3e^{-0.5x}\cos(1.323x) + c_4e^{-0.5x}\sin(1.323x)$

31. **(a)** $\{x, x^{-1}, x^2\}$ **(b)** $\{x, x^2, x^{-1}, x^{-2}\}$ **(c)** $\{x, x^2\cos(3 \ln x), x^2\sin(3 \ln x)\}$

33. **(b)** $x(t) = c_1 \cos t + c_2 \sin t + c_3 \cos \sqrt{6}t + c_4 \sin \sqrt{6}t$
(c) $y(t) = 2c_1 \cos t + 2c_2 \sin t - (c_3/2) \cos \sqrt{6}t - (c_4/2)\sin \sqrt{6}t$
(d) $x(t) = (3/5)\cos t + (2/5)\cos \sqrt{6}t$, $y(t) = (6/5)\cos t - (1/5)\cos \sqrt{6}t$

35. $c_1 \cosh rx + c_2 \sinh rx + c_3 \cos rx + c_4 \sin rx$, where $r^4 = k/(EI)$

Exercises 6.3, page 337

1. $c_1xe^x + c_2 + c_3x + c_4x^2$

3. $c_1x^2e^{-2x}$

5. $c_1e^x + c_2e^{3x} + c_3e^{-2x} - (1/6)xe^x + (1/6)x^2 + (5/18)x + 37/108$

7. $c_1e^x + c_2e^{-2x} + c_3xe^{-2x} - (1/6)x^2e^{-2x}$

9. $c_1e^x + c_2xe^x + c_3x^2e^x + (1/6)x^3e^x$

11. D^5 **13.** $D + 7$ **15.** $(D - 2)(D - 1)$

17. $[(D + 1)^2 + 4]^3$ **19.** $(D + 2)^2[(D + 5)^2 + 9]^2$

21. $c_3 \cos 2x + c_4 \sin 2x + c_5$

23. $c_3xe^{3x} + c_4x^2 + c_5x + c_6$

25. $c_3 + c_4x + c_5 \cos 2x + c_6 \sin 2x$

27. $c_3xe^{-x}\cos x + c_4xe^{-x}\sin x + c_5x^2 + c_6x + c_7$

29. $c_2x + c_3x^2 + c_6x^2e^x$

31. $-2e^{3x} + e^{-2x} + x^2 - 1$ **33.** $x^2e^{-2x} - x^2 + 3$

39. $x(t) = -(1/63)e^{3t} + c_1 + c_2t - c_3e^{\sqrt{2}t} - c_4e^{-\sqrt{2}t}$, $y(t) = (8/63)e^{3t} + c_1 + c_2t + c_3e^{\sqrt{2}t} + c_4e^{-\sqrt{2}t}$

Exercises 6.4, page 341

1. $(1/6)x^2e^{2x}$ **3.** $e^{2x}/16$

5. $\ln(\sec x) - (\sin x)\ln(\sec x + \tan x)$

7. $c_1x + c_2x^2 + c_3x^3 - (1/24)x^{-1}$

9. $-(1/2)e^x \int e^{-x}g(x)\,dx + (1/6)e^{-x} \int e^xg(x)\,dx + (1/3)e^{2x} \int e^{-2x}g(x)\,dx$

11. $c_1x + c_2x^{-1} + c_3x^3 - x \sin x - 3 \cos x + 3x^{-1} \sin x$