***Exercises 3.6, page 129***

**3.**

| $h$ | "$e$" |
|---|---|
| 1 | 3 |
| 0.1 | 2.72055 |
| 0.01 | 2.71830 |
| 0.001 | 2.71828 |
| 0.0001 | 2.71828 |

**7.**

| $x_n$ | $y_n$ |
|---|---|
| 1.1 | 0.10450 |
| 1.2 | 0.21668 |
| 1.3 | 0.33382 |
| 1.4 | 0.45300 |
| 1.5 | 0.57135 |

**9.**

| $x_n$ | $y_n$ |
|---|---|
| 0.2 | 0.61784 |
| 0.4 | 1.23864 |
| 0.6 | 1.73653 |
| 0.8 | 1.98111 |
| 1.0 | 1.99705 |
| 1.2 | 1.88461 |
| 1.4 | 1.72447 |
| 1.6 | 1.56184 |
| 1.8 | 1.41732 |
| 2.0 | 1.29779 |

**11.** $\phi(1) \approx x(1; 2^{-3}) = 1.25494$

**13.** $\phi(1) \approx y(1; 2^{-3}) = 0.71698$

**15.** $x = 1.27$

**17.**

| $x_n$ | $y_n(h = 0.2)$ | $y_n(h = 0.1)$ | $y_n(h = 0.025)$ |
|---|---|---|---|
| 0.1 | | −1 | 0.06250 |
| 0.2 | −3 | 1 | 0.00391 |
| 0.3 | | −1 | 0.00024 |
| 0.4 | 9 | 1 | 0.00002 |
| 0.5 | | −1 | 0.00000 |
| 0.6 | −27 | 1 | 0.00000 |
| 0.7 | | −1 | 0.00000 |
| 0.8 | 81 | 1 | 0.00000 |
| 0.9 | | −1 | 0.00000 |
| 1.0 | −243 | 1 | 0.00000 |

We conclude that step size can dramatically affect convergence.

**19.**

| Time | $T_n$: $K = 0.2$ | $K = 0.4$ | $K = 0.6$ |
|---|---|---|---|
| Midnight | 65.0000 | 65.0000 | 65.0000 |
| 4 A.M. | 69.1639 | 68.5644 | 68.1300 |
| 8 A.M. | 71.4836 | 72.6669 | 73.6678 |
| Noon | 72.9089 | 75.1605 | 76.9783 |
| 4 P.M. | 72.0714 | 73.5977 | 74.7853 |
| 8 P.M. | 69.8095 | 69.5425 | 69.2831 |
| Midnight | 68.3852 | 67.0500 | 65.9740 |

***Exercises 3.7, page 139***

**1.** $y_{n+1} = y_n + h\cos(x_n + y_n) - \dfrac{h^2}{2}\sin(x_n + y_n)[1 + \cos(x_n + y_n)]$

**3.** $y_{n+1} = y_n + h(x_n - y_n) + \dfrac{h^2}{2}(1 - x_n + y_n) - \dfrac{h^3}{6}(1 - x_n + y_n) + \dfrac{h^4}{24}(1 - x_n + y_n)$

**5.** Order 2, $\phi(1) \approx 1.3725$; order 4, $\phi(1) \approx 1.3679$

**7.** $-11.7679$ **9.** $1.36789$ **11.** $x = 1.41$

**13.** $x = 0.50$

**15.**

| $x_n$ | $y_n$ |
|---|---|
| 0.5 | 0.21462 |
| 1.0 | 0.13890 |
| 1.5 | −0.02668 |
| 2.0 | −0.81879 |
| 2.5 | −1.69491 |
| 3.0 | −2.99510 |

**19.** $v(3) \approx 0.24193$ with $h = 0.0625$

**21.** $z(1) \approx 2.87083$ with $h = 0.03125$

## CHAPTER 4

***Exercises 4.1, page 157***

**3.** Both approach zero. **5.** 0

**7.** $y(t) = -(30/61)\cos 3t - (25/61)\sin 3t$

**9.** $y(t) = -2\cos 2t + (3/2)\sin 2t$

***Exercises 4.2, page 165***

**1.** $c_1e^{-3t} + c_2te^{-3t}$

**3.** $c_1e^{2t} + c_2e^{-t}$

**5.** $c_1e^{2t} + c_2e^{3t}$

**7.** $c_1e^{t/2} + c_2e^{-2t/3}$

**9.** $c_1e^{t/2} + c_2te^{t/2}$

**11.** $c_1e^{-5t/2} + c_2te^{-5t/2}$

**13.** $3e^{-4t}$ **15.** $2e^{5(t+1)} + e^{-(t+1)}$

**17.** $(\sqrt{3}/2)[e^{(1+\sqrt{3})t} - e^{(1-\sqrt{3})t}]$

**19.** $e^{-t} - 2te^{-t}$

**21.** **(a)** $ar + b = 0$ **(b)** $ce^{-bt/a}$

**23.** $ce^{-4t/5}$

**25.** $ce^{13t/6}$

**27.** Lin. dep. **29.** Lin. indep. **31.** Lin. dep.

**33.** If $c_1 \neq 0$, then $y_1 = -(c_2/c_1)y_2$ .

**35.** **(a)** Lin. indep. **(b)** Lin. dep. **(c)** Lin. indep. **(d)** Lin. dep.

**37.** $c_1e^t + c_2e^{(-1-\sqrt{5})t} + c_3e^{(-1+\sqrt{5})t}$

**39.** $c_1e^{-2t} + c_2te^{-2t} + c_3e^{2t}$

**41.** $c_1e^{-3t} + c_2e^{-2t} + c_3e^{2t}$

**43.** $3 + e^t - 2e^{-t}$

**45.** **(a)** $c_1e^{r_1t} + c_2e^{r_2t} + c_3e^{r_3t}$ (where $r_1 = -4.832$, $r_2 = -1.869$, and $r_3 = 0.701$)

**(b)** $c_1e^{r_1t} + c_2e^{-r_1t} + c_3e^{r_2t} + c_4e^{-r_2t}$ (where $r_1 = 1.176$, $r_2 = 1.902$)

**(c)** $c_1e^{-t} + c_2e^t + c_3e^{-2t} + c_4e^{2t} + c_5e^{3t}$

### Exercises 4.3, page 173

**1.** $c_1 \cos t + c_2 \sin t$
**3.** $c_1 e^{5t} \cos t + c_2 e^{5t} \sin t$
**5.** $c_1 e^{2t} \cos(\sqrt{3}t) + c_2 e^{2t} \sin(\sqrt{3}t)$
**7.** $c_1 e^{-t/2} \cos(\sqrt{5}t/2) + c_2 e^{-t/2} \sin(\sqrt{5}t/2)$
**9.** $c_1 e^{-2t} \cos 2t + c_2 e^{-2t} \sin 2t$
**11.** $c_1 e^{-5t} + c_2 t e^{-5t}$
**13.** $c_1 e^{-t} \cos 2t + c_2 e^{-t} \sin 2t$
**15.** $c_1 e^{-5t} \cos 4t + c_2 e^{-5t} \sin 4t$
**17.** $c_1 e^{t/2} \cos(3\sqrt{3}t/2) + c_2 e^{t/2} \sin(3\sqrt{3}t/2)$
**19.** $c_1 e^{t} + c_2 e^{-t} \cos 2t + c_3 e^{-t} \sin 2t$
**21.** $2e^{-t} \cos t + 3e^{-t} \sin t$
**23.** $(\sqrt{2}/4)[e^{(2+\sqrt{2})t} - e^{(2-\sqrt{2})t}]$
**25.** $e^{t} \sin t - e^{t} \cos t$
**27.** $e^{2t} - \sqrt{2}\, e^{t} \sin \sqrt{2}t$
**29.** **(a)** $c_1 e^{-t} + c_2 e^{t} \cos \sqrt{2}t + c_3 e^{t} \sin \sqrt{2}t$
**(b)** $c_1 e^{2t} + c_2 e^{-2t} \cos 3t + c_3 e^{-2t} \sin 3t$
**(c)** $c_1 \cos 2t + c_2 \sin 2t + c_3 \cos 3t + c_4 \sin 3t$
**31.** **(a)** Oscillatory **(b)** Tends to zero
**(c)** Tends to $-\infty$ **(d)** Tends to $-\infty$
**(e)** Tends to $+\infty$
**33.** **(a)** $y(t) = 0.3e^{-3t} \cos 4t + 0.2e^{-3t} \sin 4t$
**(b)** $2/\pi$
**(c)** Decreases the frequency of oscillation, introduces the factor $e^{-3t}$, causing the solution to decay to zero
**35.** $b \geq 2\sqrt{Ik}$
**37.** **(a)** $c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t$
**(b)** $(c_1 + c_2 t)e^{-t} \cos(\sqrt{3}t) + (c_3 + c_4 t)e^{-t} \sin(\sqrt{3}t)$

### Exercises 4.4, page 181

**1.** No **3.** Yes **5.** Yes
**7.** No, not a constant coefficient equation
**9.** $y_p(t) \equiv -10$ **11.** $y_p(x) = [(\ln 2)^2 + 1]^{-1} 2^x$
**13.** $\cos 3t$
**15.** $xe^x/2 + 3e^x/4$ **17.** $4t^2 e^t$
**19.** $\left(\frac{t}{13} + \frac{8}{169}\right) t e^{-3t}$ **21.** $t^3 e^{2t}/6$
**23.** $-\frac{1}{21}\theta^3 - \frac{1}{49}\theta^2 - \frac{2}{343}\theta$
**25.** $e^{2t}(\cos 3t + 6 \sin 3t)$
**27.** $(A_3 t^4 + A_2 t^3 + A_1 t^2 + A_0 t) \cos 3t$
$+ (B_3 t^4 + B_2 t^3 + B_1 t^2 + B_0 t) \sin 3t$
**29.** $e^{3t}(A_6 t^8 + A_5 t^7 + A_4 t^6 + A_3 t^5 + A_2 t^4 + A_1 t^3 + A_0 t^2)$
**31.** $(A_3 t^4 + A_2 t^3 + A_1 t^2 + A_0 t)e^{-t} \cos t$
$+ (B_3 t^4 + B_2 t^3 + B_1 t^2 + B_0 t)e^{-t} \sin t$
**33.** $(1/5) \cos t + (2/5) \sin t$
**35.** $\left(\frac{1}{10}t^2 - \frac{4}{25}t\right)e^t$

### Exercises 4.5, page 187

**1.** **(a)** $t/4 - 1/8 + [\sin(2t)]/4$
**(b)** $t/2 - 1/4 - (3/4)\sin(2t)$
**(c)** $11t/4 - 11/8 - 3\sin(2t)$
**3.** $y = t + c_1 + c_2 e^{-t}$
**5.** $y = e^x + x^2 + c_1 e^{-2x} + c_2 e^{-3x}$
**7.** $y = \tan x + c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}$
**9.** Yes **11.** No **13.** Yes **15.** No
**17.** $y = 11t - 1 + c_1 e^t + c_2 e^{-t}$
**19.** $y = (\cos x - \sin x)e^x/2 + c_1 e^x + c_2 e^{2x}$
**21.** $y = (1/2)\theta e^{-\theta} \sin\theta + (c_1 \cos\theta + c_2 \sin\theta)e^{-\theta}$
**23.** $y = e^t - 1$
**25.** $z = e^{-x} - \cos x + \sin x$
**27.** $y = -(3/10)\cos x - (1/10)\sin x - (1/20)\cos 2x + (3/20)\sin 2x$
**29.** $y = -(1/2)\sin\theta - (1/3)e^{2\theta} + (3/4)e^{\theta} + (7/12)e^{-\theta}$
**31.** $y_p = (A_1 t + A_0)t \cos t + (B_1 t + B_0)t \sin t + C \cdot 10^t$
**33.** $x_p(t) = (A\cos t + B\sin t)e^t + C_2 t^2 + C_1 t + C_0$
$+ D_1 \cos 3t + D_2 \sin 3t + E_1 \cos t + E_2 \sin t$
**35.** $y_p = (A_1 t + A_0)\cos 3t + (B_1 t + B_0)\sin 3t + Ce^{5t}$
**37.** $y_p = t^2 + 3t - 1$
**39.** $y_p = (t/10 - 4/25)te^t - 1/2$
**41.** **(a)** $y_1 = -(2\cos 2t + \sin 2t)e^{-t} + 2$ for $0 \leq t \leq 3\pi/2$
**(b)** $y_2 = y_h = (c_1 \cos 2t + c_2 \sin 2t)e^{-t}$ for $t > 3\pi/2$
**(c)** $c_1 = -2(e^{3\pi/2} + 1)$ , $c_2 = -(e^{3\pi/2} + 1)$
**43.** $y = -\cos t + (1/2)\sin t - (1/2)e^{-3t} + 2e^{-t}$
**45.** **(a)** $y(t) = \dfrac{2V\cos(\pi/2V)}{V^2 - 1}\sin t$ for $V \neq 1$;
$y(t) = \dfrac{\pi}{2}\sin t$ for, $V = 1$
**(b)** $V \approx 0.73$
**47.** **(a)** $2\sin 3t - \cos 6t$ **(b)** No solution
**(c)** $c\sin 3t - \cos 6t$ , where $c$ is any constant

### Exercises 4.6, page 192

**1.** $(\cos t)\ln|\cos t| + t\sin t + c_1 \cos t + c_2 \sin t$
**3.** $t^2 e^{-t}/2 + c_1 e^{-t} + c_2 t e^{-t}$
**5.** $-(1/9) + (1/9)(\sin 3t)\ln|\sec 3t + \tan 3t|$
$+ c_1 \cos 3t + c_2 \sin 3t$
**7.** $(2\ln t - 3)t^2 e^{-2t}/4 + c_1 e^{-2t} + c_2 t e^{-2t}$
**9.** $y_p = -2t - 4$
**11.** $c_1 \cos t + c_2 \sin t + (\sin t)\ln|\sec t + \tan t| - 2$
**13.** $c_1 \cos 2t + c_2 \sin 2t + (1/24)\sec^2 2t - 1/8$
$+ (1/8)(\sin 2t)\ln|\sec 2t + \tan 2t|$
**15.** $c_1 \cos t + c_2 \sin t - t^2 + 3 + 3t\sin t$
$+ 3(\cos t)\ln|\cos t|$
**17.** $c_1 \cos 2t + c_2 \sin 2t - e^t/5$
$-(1/2)(\cos 2t)\ln|\sec 2t + \tan 2t|$

**19.** $y = e^{1-t} - e^{t-1} + \frac{e^t}{2}\int_1^t \frac{e^{-u}}{u}\,du - \frac{e^{-t}}{2}\int_1^t \frac{e^{u}}{u}\,du$

$[y(2) \approx -1.93]$

**21.** 0.3785

## Exercises 4.7, page 200

**1.** Unique solution on (0, 3)
**3.** Unique solution on $(0, \infty)$
**5.** Does not apply; $t = 0$ is a point of discontinuity
**7.** Does not apply; not an initial value problem
**9.** $c_1t^{-3} + c_2t^2$
**11.** $c_1t^{-1} + c_2t^{-4}$
**13.** $c_1t^{-1/3} + c_2t^{-1/3}\ln t$
**15.** $c_1t\cos[2\ln(-t)] + c_2t\sin[2\ln(-t)]$
**17.** $t^{-4}\{c_1\cos[\ln(-t)] + c_2\sin[\ln(-t)]\}$
**19.** $t - 3t^4$
**21.** $c_1(t-2) + c_2(t-2)^7$
**23.** **(c)** $t^\alpha\cos(\beta\ln|t|)$, $t^\alpha\sin(\beta\ln|t|)$; $t^r$, $t^r\ln|t|$
**25.** **(a)** True **(b)** False
**27.** **(e)** No, because the coefficient of $y''$ vanishes at $t = 0$ and the equation cannot be written in standard form.
**29.** Otherwise their Wronskian would be zero at $t_0$, contradicting linear independence.
**31.** **(a)** Yes **(b)** No **(c)** Yes **(d)** Yes
**33.** $Cte^{-t}$
**35.** $1 + 2t - t^2$
**37.** $c_1e^t + c_2(t+1) - t^2$
**39.** $c_1(5t-1) + c_2e^{-5t} - t^2e^{-5t}/10$
**41.** $c_1\cos(3\ln t) + c_2\sin(3\ln t)$
$+ (1/9)\cos(3\ln t)\ln|\sec(3\ln t) + \tan(3\ln t)|$
**43.** $c_1t + c_2t\ln t + (1/2)t(\ln t)^2 + 3t(\ln t)[\ln|\ln t|]$
**45.** $t^4$
**47.** $t + 1$
**49.** **(a)** $(1-2t^2)\int(1-2t^2)^{-2}e^{t^2}dt$

**(b)** $(3t-2t^3)\int(3t-2t^3)^{-2}e^{t^2}dt$

**51.** $tw'' + 2tw' + (t+1)w = 0$
**53.** **(a)** $\phi'(t_0) = \lim_{n\to\infty}\frac{\phi(t_n) - \phi(t_0)}{t_n - t_0} = \lim_{n\to\infty}\frac{0-0}{t_n - t_0} = 0$

## Exercises 4.8, page 212

**1.** Let $Y(t) = y(-t)$. Then $Y'(t) = -y'(-t)$, $Y''(t) = y''(-t)$. But $y''(s) - sy(s) = 0$, so $y''(-t) + ty(-t) = 0$ or $Y''(t) + tY(t) = 0$.
**3.** The spring stiffness is $(-6y)$, so it opposes negative displacements $(y < 0)$ and reinforces positive displacements $(y > 0)$. Initially $y < 0$ and $y' < 0$, so the (positive) stiffness reverses the negative velocity and restores $y$ to 0. Thereafter, $y > 0$ and the negative stiffness drives $y$ to $+\infty$.
**5.** **(a)** $y'' = 2y^3 = \frac{d}{dy}(y^4/2)$. Thus, by setting $K = 0$ and choosing the $(-)$ sign in equation (11), we get

$$t = -\int\frac{dy}{\sqrt{2y^4/2}} + c = \frac{1}{y} + c, \text{ or } y = 1/(t-c) .$$

**(b)** Linear dependence would imply

$$\frac{y_1(t)}{y_2(t)} = \frac{1/(t-c_1)}{1/(t-c_2)} = \frac{t-c_2}{t-c_1} \equiv \text{constant}$$

in a neighborhood of 0, which is false if $c_1 \neq c_2$.
**(c)** If $y(t) = 1/(t-c)$, then $y(0) = -1/c$, $y'(0) = -1/c^2 = -y(0)^2$, which is false for the given data.
**7.** **(a)** The velocity, which is always perpendicular to the lever arm, is $\ell\, d\theta/dt$. Thus, (lever arm) times (perpendicular momentum) $= \ell m\ell\, d\theta/dt = m\ell^2 d\theta/dt$.
**(b)** The component of the gravitational force perpendicular to lever arm is $mg\sin\theta$, and is directed toward decreasing $\theta$. Thus, torque $= -\ell mg\sin\theta$.
**(c)** Torque $= \frac{d}{dt}$ (angular momentum) or
$-\ell mg\sin\theta = (m\ell^2\theta')' = m\ell^2\theta''$ .
**9.** 2 or $-2$
**11.** The sign of the damping coefficient $(y')^2 - 1$ indicates that low velocities are boosted by negative damping but that high velocities are slowed. Hence, one expects a limit cycle.
**13.** **(a)** Airy **(b)** Duffing **(c)** van der Pol
**15.** **(a)** Yes ($t^2$ = positive stiffness)
**(b)** No ($-t^2$ = negative stiffness)
**(c)** Yes ($y^4$ = positive stiffness)
**(d)** No ($y^5$ = negative stiffness for $y < 0$)
**(e)** Yes ($4 + 2\cos t$ = positive stiffness)
**(f)** Yes (positive stiffness and damping)
**(g)** No (negative stiffness and damping)
**17.** $1/4\sqrt{2}$

## Exercises 4.9, page 222

**1.** $y(t) = -(1/4)\cos 5t - (1/5)\sin 5t$
amplitude $= \sqrt{41}/20$; period $= 2\pi/5$;
frequency $= 5/2\pi$; $[\pi - \arctan(5/4)]/5$ sec
**3.** $b = 0$: $y(t) = \cos 4t$

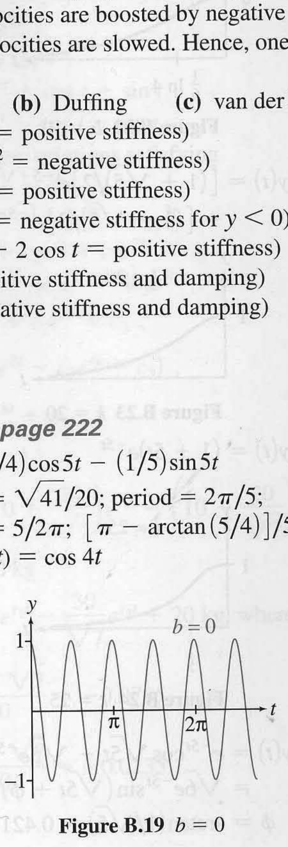


**Figure B.19** $b = 0$

$b = 6$: $y(t) = e^{-3t}\cos\sqrt{7}t + (3/\sqrt{7})e^{-3t}\sin\sqrt{7}t$
$= (4/\sqrt{7})e^{-3t}\sin(\sqrt{7}t + \phi)$, where
$\phi =$ arctan $\sqrt{7}/3 \approx 0.723$

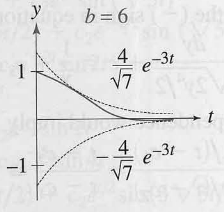


**Figure B.20** $b = 6$

$b = 8$: $y(t) = (1 + 4t)e^{-4t}$

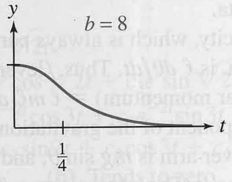


**Figure B.21** $b = 8$

$b = 10$: $y(t) = (4/3)e^{-2t} - (1/3)e^{-8t}$

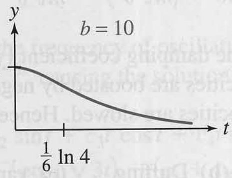


**Figure B.22** $b = 10$

**5.** $k = 20$: $y(t) = [(1 + \sqrt{5})/2]e^{(-5+\sqrt{5})t} + [(1 - \sqrt{5})/2]e^{(-5-\sqrt{5})t}$

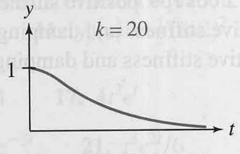


**Figure B.23** $k = 20$

$k = 25$: $y(t) = (1 + 5t)e^{-5t}$

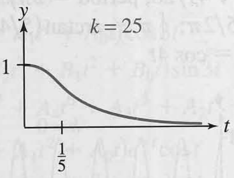


**Figure B.24** $k = 25$

$k = 30$: $y(t) = e^{-5t}\cos\sqrt{5}t + \sqrt{5}e^{-5t}\sin\sqrt{5}t$
$= \sqrt{6}e^{-5t}\sin(\sqrt{5}t + \phi)$, where
$\phi = \arctan(1/\sqrt{5}) \approx 0.421$

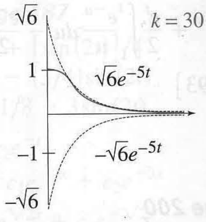


**Figure B.25** $k = 30$

**7.** $y(t) = (-3/4)e^{-8t}\cos 8t - e^{-8t}\sin 8t$
$= (5/4)e^{-8t}\sin(8t + \phi)$ where
$\phi = \pi +$ arctan$(3/4) \approx 3.785$ ;
damp. factor $= (5/4)e^{-8t}$ ;
quasiperiod $= \pi/4$; quasifreq. $= 4/\pi$

**9.** 0.242 m

**11.** $(10/\sqrt{9999})$ arctan$(\sqrt{9999}) \approx 0.156$ sec

**13.** Relative extrema at
$t = [\pi/3 + n\pi -$ arctan$(\sqrt{3}/2)]/(2\sqrt{3})$
for $n = 0, 1, 2, \ldots$; but touches curves $\pm\sqrt{7/12}e^{-2t}$
at $t = [\pi/2 + m\pi$ arctan$(\sqrt{3}/2)]/(2\sqrt{3})$
for $m = 0, 1, 2, \ldots$

**15.** First measure half the quasiperiod $P$ as the time between two successive zero crossings. Then compute the ratio $y(t + P)/y(t) = e^{-(b/2m)P}$.

### *Exercises 4.10, page 229*

**1.** $M(\gamma) = 1/\sqrt{(1 - 4\gamma^2)^2 + 4\gamma^2}$

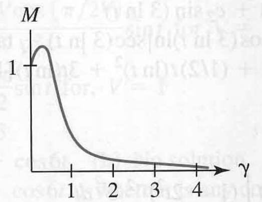


**Figure B.26**

**3.** $y(t) = \cos 3t + (1/3)t\sin 3t$

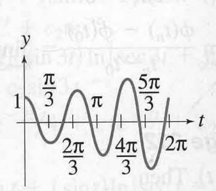


**Figure B.27**

**5. (a)** $y(t) = -[F_0/(k - m\gamma^2)]\cos(\sqrt{k/m}\,t)$
$+ [F_0/(k - m\gamma^2)]\cos\gamma t$
$= (F_0/[m(\omega^2 - \gamma^2)])(\cos\gamma t - \cos\omega t)$

**(c)** $y(t) = \sin 8t \sin t$

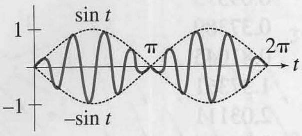


**Figure B.28**

**7.** $y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} + \dfrac{F_0 \sin(\gamma t + \theta)}{\sqrt{(k - m\gamma^2)^2 + b^2\gamma^2}}$,
where $r_1, r_2 = -(b/2m) \pm (1/2m)\sqrt{b^2 - 4mk}$ and $\tan\theta = (k - m\gamma^2)/(b\gamma)$ as in equation (7)

**9.** $y_p(t) = (0.08)\cos 2t + (0.06)\sin 2t$
$= (0.1)\sin(2t + \theta)$ , where
$\theta = \arctan(4/3) \approx 0.927$

**11.** $y(t) = -(18/85)e^{-2t}\cos 6t - (22/255)e^{-2t}\sin 6t + (2/\sqrt{85})\sin(2t + \theta)$ , where
$\theta = \arctan(9/2) \approx 1.352$ ;
res. freq. $= 2\sqrt{2}/\pi$ cycles/sec

**13.** $y_p(t) = (3/185)(8\sin 4t - 11\cos 4t)$

**15.** $\text{Amp} = \sqrt{\left(-\dfrac{11}{986}\right)^2 + \left(-\dfrac{3}{1972}\right)^2} \approx 0.01(\text{m})$ ,
freq. $= 2/\pi$

### *Review Problems, page 233*

**1.** $c_1 e^{-9t} + c_2 e^{t}$

**3.** $c_1 e^{t/2}\cos(3t/2) + c_2 e^{t/2}\sin(3t/2)$

**5.** $c_1 e^{3t/2} + c_2 e^{t/3}$

**7.** $c_1 e^{-t/3}\cos(t/6) + c_2 e^{-t/3}\sin(t/6)$

**9.** $c_1 e^{7t/4} + c_2 t e^{7t/4}$

**11.** $t^{1/2}\{c_1\cos[(\sqrt{19}/2)\ln t] + c_2\sin[(\sqrt{19}/2)\ln t]\}$

**13.** $c_1\cos 4t + c_2\sin 4t + (1/17)te^{t} - (2/289)e^{t}$

**15.** $c_1 e^{-2t} + c_2 e^{-t} + c_3 e^{-t/3}$

**17.** $c_1 e^{t} + c_2 e^{-t/2}\cos(\sqrt{43}t/2) + c_3 e^{-t/2}\sin(\sqrt{43}t/2)$

**19.** $c_1 e^{-3t} + c_2 e^{t/2} + c_3 t e^{t/2}$

**21.** $c_1 e^{3t/2}\cos(\sqrt{19}t/2) + c_2 e^{3t/2}\sin(\sqrt{19}t/2) - e^{t}/5 + t^2 + 6t/7 + 4/49$

**23.** $c_1\cos 4\theta + c_2\sin 4\theta - (1/16)(\cos 4\theta)\ln|\sec 4\theta + \tan 4\theta|$

**25.** $c_1 e^{3t/2} + c_2 t e^{3t/2} + e^{3t}/9 + e^{5t}/49$

**27.** $c_1 x + c_2 x^{-2} - 2x^{-2}\ln x + x\ln x$

**29.** $e^{-2t}\cos(\sqrt{3}t)$

**31.** $2e^{t}\cos 3t - (7/3)e^{t}\sin 3t - \sin 3t$

**33.** $-e^{-t} - 3e^{5t} + e^{8t}$

**35.** $\cos\theta + 2\sin\theta + \theta\sin\theta + (\cos\theta)\ln|\cos\theta|$

**37.** **(a)**, **(c)**, **(e)**, and **(f)** have all solutions bounded as $t \to +\infty$

**39.** $y_p(t) = (1/4)\sin 8t$ ; $\sqrt{62}/2\pi$

## CHAPTER 5

### *Exercises 5.2, page 250*

**1.** **(a)** $-t^3 + 3t^2 + 8$ **(b)** $-2t^3 + 3t^2 + 6t + 16$
**(c)** $2t^3 + 3t^2 - 16$ **(d)** $-2t^3 + 3t^2 + 6t + 16$
**(e)** $-2t^3 + 3t^2 + 6t + 16$

**3.** $x = c_1 + c_2 e^{-2t}$ ; $y = c_2 e^{-2t}$

**5.** $x \equiv -5$ ; $y \equiv 1$

**7.** $u = c_1 - (1/2)c_2 e^{-t} + (1/2)e^{t} + (5/3)t$ ;
$v = c_1 + c_2 e^{-t} + (5/3)t$

**9.** $x = c_1 e^{t} + (1/4)\cos t - (1/4)\sin t$ ;
$y = -3c_1 e^{t} - (3/4)\cos t - (1/4)\sin t$

**11.** $u = c_1\cos 2t + c_2\sin 2t + c_3 e^{\sqrt{3}t} + c_4 e^{-\sqrt{3}t} - (3/10)e^{t}$ ;
$v = c_1\cos 2t + c_2\sin 2t - (2/5)c_3 e^{\sqrt{3}t} - (2/5)c_4 e^{-\sqrt{3}t} + (1/5)e^{t}$

**13.** $x = 2c_2 e^{t}\cos 2t - 2c_1 e^{t}\sin 2t$ ;
$y = c_1 e^{t}\cos 2t + c_2 e^{t}\sin 2t$

**15.** $w = -(2/3)c_1 e^{2t} + c_2 e^{7t} + t + 1$ ;
$z = c_1 e^{2t} + c_2 e^{7t} - 5t - 2$

**17.** $x = c_1\cos t + c_2\sin t - 4c_3\cos(\sqrt{6}t) - 4c_4\sin(\sqrt{6}t)$ ;
$y = c_1\cos t + c_2\sin t + c_3\cos(\sqrt{6}t) + c_4\sin(\sqrt{6}t)$

**19.** $x = 2e^{3t} - e^{2t}$ ;
$y = -2e^{3t} + 2e^{2t}$

**21.** $x = e^{t} + e^{-t} + \cos t + \sin t$ ;
$y = e^{t} + e^{-t} - \cos t - \sin t$

**23.** Infinitely many solutions satisfying
$x + y = e^{t} + e^{-2t}$

**25.** $x(t) = -c_1 e^{t} - 2c_2 e^{2t} - c_3 e^{3t}$ ,
$y(t) = c_1 e^{t} + c_2 e^{2t} + c_3 e^{3t}$ ,
$z(t) = 2c_1 e^{t} + 4c_2 e^{2t} + 4c_3 e^{3t}$

**27.** $x(t) = c_1 e^{8t} + c_2 e^{4t} + c_3$ ,
$y(t) = \dfrac{1}{2}(c_1 e^{8t} - c_2 e^{4t} + c_3)$ ,
$z(t) = -c_1 e^{8t} + c_3$

**29.** $-3 \le \lambda \le -1$

**31.** $x(t) = -\left(10 + \dfrac{20}{\sqrt{7}}\right)e^{r_1 t} - \left(10 - \dfrac{20}{\sqrt{7}}\right)e^{r_2 t} + 20$ kg ,
$y(t) = \dfrac{30}{\sqrt{7}}e^{r_1 t} - \dfrac{30}{\sqrt{7}}e^{r_2 t} + 20$ kg, where
$r_1 = \dfrac{-5 - \sqrt{7}}{100} \approx -0.0765$ ,
$r_2 = \dfrac{-5 + \sqrt{7}}{100} \approx -0.0235$