## MA 341-001 Test 3

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Use your own paper to work the problems. On all problems, you must **show your work** to receive credit.

## Don't do more than each question asks you to do.

When you finish, fold this paper lengthwise together with your work, so that this writing is on the outside. Write your name and row number above (the front row is row 1), and turn in.

1. Consider the differential equation

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \mathbf{x}(t)$$

Let

$$\mathbf{x}_1(t) = \begin{bmatrix} 2e^{4t} \\ 3e^{4t} \end{bmatrix}$$
 and  $\mathbf{x}_2(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ 

- (a) Check that  $\mathbf{x}_1(t)$  is a solution. (Just substitute and check that it works.)
- (b)  $\mathbf{x}_2(t)$  is also a solution. (You don't need to check this.) Show that  $\mathbf{x}_1(t)$  and  $\mathbf{x}_2(t)$  are linearly independent.
- (c) Give the general solution.
- (d) Give the solution that satisfies the initial condition

$$\mathbf{x}(0) = \begin{bmatrix} 0\\ -5 \end{bmatrix}.$$

2. Find the eigenvalues of the following matrix. If there is a repeated eigenvalue, list it twice.

$$\mathbf{A} = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

3. The matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{bmatrix}$$

has the eigenvalues r = -1, 1, 2. Eigenvectors for the eigenvalues -1 and 1 are

$$\begin{bmatrix} 1\\ -1\\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1\\ 1\\ 1 \end{bmatrix}$$

respectively.

- (a) Find an eigenvector for the eigenvalue 2.
- (b) Give the general solution of  $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$ .
- 4. The matrix

$$\mathbf{A} = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$$

has the eigenvalues  $r = 1 \pm 2i$ . Find the general solution of  $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$ .

5. Consider the system

$$\frac{dx}{dt} = -x,$$
$$\frac{dy}{dt} = -3y$$

- (a) Solve the phase plane equation (i.e., the equation for  $\frac{dy}{dx}$ .
- (b) Sketch some representative trajectories with their flow arrows. Your sketch should clearly show any equilibria.