MA 341-003 Test 1 Review Questions

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Warning: Not everything is covered.

1. Translate into a differential equation: Sec. 1.1 problems 15 and 16.
2. Checking whether something is a solution of a differential equation: Sec. 1.2 problems 8 and 10.
3. Existence-uniqueness theorem: Sec. 1.2 problem 26. Also: What if the initial condition is $y(1) = 2$?
4. Direction fields: Sec. 1.3 problem 17. Use the method of isoclines to sketch the direction field in the region $x > 0$. Then try to sketch the solutions with $y(1) = 1$ and $y(1) = 5$. For these two solutions, as $x \to \infty$, what do you think y approaches?
5. Euler's method: Sec. 1.4 problem 6. Just do the points $x = 1.1$, 1.2. Don't round.
6. Separable equations: Sec. 2.2 problem 26.
7. Linear equations: Sec. 2.3 problems 16, 20.
8. Exact equations: Sec. 2.4 problem 12.
9. Mixing: Sec. 3.2 problem 2.
10. Mechanics: Sec. 3.4 problem 6. Initially the object is 100 m above the ground. First draw your coordinate axis!

Answers:

1. 16: $\frac{dA}{dt} = kA^2$.
2. Both are solutions. Just calculate $\frac{dy}{dx}$ and plug into the differential equation.
3. No, yes. Look at $\frac{\partial f}{\partial y}$ at the two points.
4. Suggestion: Look at the isoclines $c = -2$, -1, 0, 1, 2.
5. Approximation from Euler's method: At $x = 1.1$, $y = 0.1$; at $x = 1.2$, $y = 0.209$.
6. $y = \left(1 - \frac{1}{2}\ln(1+x)\right)^2$
7. 16: $y = \frac{1}{(x^2+1)^2}\left(\frac{x^5}{5} + \frac{x^4}{2} + x^2 - x + C\right)$

   20: $y = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{C}{x^3}$
8. $e^x \sin y - x^3 + y^{\frac{1}{3}} = C$
9. $x = 2.5 - 2.0e^{-.12t}$; t $= \frac{25}{3}\ln 2 = 5.776$.
10. If x increases as you go up, $v = -4.9 + 24.9e^{-2t}$.

    If the initial position of the object is $x = 0$, $x = -4.9t - 12.45e^{-2t} + 12.45$.

    Then $x = -100$ when (after simplifying) $22.95 = t + 2.54e^{-2t}$.

    Solution according to Maple: $t = 22.95$. (Not surprising.)