## MA 341-001 Test 1

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Use your own paper to work the problems. On all problems, you must **show your work** to receive credit.

When you finish, fold this paper lengthwise together with your work, so that this writing is on the outside. Write your name and row number above (the front row is row 1), and turn in.

1. Consider the differential equation

$$\frac{dy}{dx} = \frac{y^3}{4 - 2xy^2}$$

(a) Show that in the region y > 0, the equation

$$4\ln y - xy^2 = C$$

is an implicit solution.

(b) Find the value of C for which this equation gives a solution to the initial value problem

$$\frac{dy}{dx} = \frac{y^3}{4 - 2xy^2}, \quad y(2) = 1.$$

- (c) Does the Existence-Uniqueness Theorem guarantee that this initial value problem has a unique solution? Explain very briefly.
- 2. Use Euler's method  $y_{n+1} = y_n + hf(x_n, y_n)$  with step size h = 0.1 to approximate the solution to the initial value problem

$$\frac{dy}{dx} = 2x + y^2, \quad y(1) = 0$$

at x = 1.1 and 1.2.

3. Find the general solution of the separable differential equation

$$\frac{dy}{dx} = \frac{\sqrt{y+1}}{2x+1}$$

in the region  $x > -\frac{1}{2}$ . Give your answer with y an explicit function of x if possible.

4. Find the general solution of the following linear differential equation:

$$x^2\frac{dy}{dx} + 4xy = 1$$

Give your answer with y an explicit function of x if possible.

5. Consider the following differential equation:

$$(2y+1)\cos 2x\,dx + (y^2 + \sin 2x)\,dy = 0.$$

- (a) Show that it is exact.
- (b) Find the general solution.
- 6. Blood flows into the pancreas at 3 cm<sup>3</sup>/sec and flows out at the same rate. The blood flowing in carries a cancer drug in the concentration 0.2 g/cm<sup>3</sup>. The pancreas contains 120 cm<sup>3</sup> of blood. Initially the concentration of the drug in the pancreas is 0.1 g/cm<sup>3</sup>.
  - (a) Let x denote the number of grams of the drug in the pancreas after t seconds. Give an equation for  $\frac{dx}{dt}$ . Do not solve your differential equation.
  - (b) What is the initial condition for your differential equation?

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