MA 242-010 Test 2

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1. Consider the space curve $\mathbf{r}(t) =< \frac{1}{2}t^2, \sin t, \cos t >$.
   (a) Find the velocity vector $\mathbf{v}(t)$.
   (b) Find the speed $v(t)$.
   (c) Find the acceleration vector $\mathbf{a}(t)$.
   (d) Find the tangential component of acceleration $a_T(t)$.
   (e) Find the normal component of acceleration $a_N(t)$.

2. Let $f(x, y) = \sqrt{x + y}$.
   (a) Find and sketch the domain.
   (b) Sketch the level curves $f(x, y) = 1$ and $f(x, y) = 2$. What is their shape?

3. Suppose that $f(2, 3) = 5$, $\frac{\partial f}{\partial x}(2, 3) = -3$, and $\frac{\partial f}{\partial y}(2, 3) = 7$. Use this information to estimate $f(1.9, 3.2)$.

4. A conical pile of sand has height h feet and radius r feet. Hence its volume is $V = \frac{1}{3}\pi r^2 h$ cubic feet. More sand is being dumped onto the pile. At a certain instant, the height of the pile is 8 feet, the radius is 6 feet, the height is increasing at $\frac{1}{2}$ foot per minute, and the radius is increasing at 1 foot per minute. Use the Chain Rule for functions of several variables to determine how fast the volume is increasing at that instant.

5. Let $f(x, y, z) = x^2 \sin yz$.

   (a) Find the gradient of f at $(3, 1, \frac{\pi}{6})$.

   (b) Find an equation for the tangent plane to the surface $x^2 \sin yz = \frac{9}{2}$ at $(3, 1, \frac{\pi}{6})$.

6. Find all points (x, y) where the function $f(x, y) = xy - x^2y - y^2$ has a local minimum, a local maximum, or a saddle point. (Hint: there are two such points.) Use second partial derivatives to decide which is which.