MA 242-010 Test 1

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1. Describe in words the region of space represented by the inequality $x^2 + y^2 + z^2 + 8z \leq 9$.

2. Consider the following vectors.

   (a) Find $\mathbf{a} \cdot \mathbf{b}$.

   (b) Find $|\mathbf{a} \times \mathbf{b}|$.

   (c) Does $\mathbf{a} \times \mathbf{b}$ point up (toward the ceiling) or down (toward the floor)?

3. Consider the three points $(1, -4, 0)$, $(5, 2, 3)$ and $(-2, -4, 1)$.

   (a) Find the equation of the plane that passes through these three points.

   (b) Find the area of the triangle whose vertices are these three points.

4. Find the parametric equations of the line that passes through $(2, 0, -1)$ and is perpendicular to the plane $x - 5y = 3$.

5. Find any points of intersection of the line

$$x = 2,\ y = 1 - t,\ z = 1 + 2t$$

and the surface

$$z = x^2 + y^2.$$

Remember, your answer should be *points* (x, y, z).

6. Consider the surface $z = 4x^2 + 9y^2$.

   (a) Find the trace in the xz-plane. Identify the curve (circle, ellipse, hyperbola, parabola, ...).

   (b) Find the trace in the yz-plane. Identify the curve.

   (c) Find the traces in planes $z = k$. Identify the curves. Consider separately $z > 0$, $z = 0$, and $z < 0$.

   (d) Sketch the surface.

7. Consider the space curve $\mathbf{r}(t) =< t, -t, t^2 >$.

   (a) Show that this curve lies in the plane $x + y = 0$.

   (b) Show that this curve lies in the surface $z = x^2$.

   (c) Make a sketch that shows the plane $x + y = 0$ and the curve $\mathbf{r}(t)$.

   (d) Find $\mathbf{r}'(t)$.