MA 242-005 Final Examination

S. Schecter

May 8, 1998

This exam is in two parts. Please do not do Part I problems and Part II problems on the same paper. When you finish, fold the Part I questions together with your Part I answers, fold the Part II questions together with your Part II answers, put your name on each part, and turn in.

Part I

1. Find the equation of the plane that includes both the point $(4, 2, 3)$ and the line
$$x = 2t, y = -1 + t, z = 3 - t.$$

2. Consider the space curve $\mathbf{r}(t) =< 2\cos t, 3\sin t, 4 >$, $0 \le t \le \pi$.

   (a) True or false: This curve lies in the plane $z = 4$.

   (b) True or false: This curve lies in the elliptic cylinder $9x^2+4y^2 = 36$.

   (c) Make a sketch that shows the plane $z = 4$ and the curve $\mathbf{r}(t)$. Indicate the direction of increasing t.

   (d) Find the velocity vector $\mathbf{v}(t)$.

   (e) Find $\mathbf{v}(\pi/2)$, the velocity vector at time $t = \pi/2$, and add it to your sketch, with its tail at the appropriate point.

   (f) Find the speed $v(t)$.

   (g) Find the acceleration vector $\mathbf{a}(t)$.

3. The value of the function $f(x, y) = e^{2x} \cos 3y$ at $(x, y) = (0, 0)$ is easy to compute. Use partial derivatives to approximate the value of this function at $(x, y) = (-0.2, 0.1)$

4. Let $f(x, y, z) = e^{x^2 y} + 2\sin(y - z)$.

   (a) Calculate f_x, f_y, and f_z.

   (b) Find the gradient of f at $(1, 0, \pi)$.

   (c) Find the directional derivative of f at $(1, 0, \pi)$ in the direction of the vector $< 2, -1, 1 >$.

   (d) Find an equation for the tangent plane to the surface $e^{x^2 y} + 2\sin(y - z) = 1$ at $(1, 0, \pi)$.

5. Let $f(x, y) = 6x^3 - 6xy + y^2$. Find all critical points of f, and determine if each is a local minimum, a local maximum, or a saddle point.

Part II

1. Evaluate

$$\int\int\int_E \frac{e^{x^2+y^2}}{x^2+y^2}\, dV$$

where E is the solid region that lies between the cylinders $x^2+y^2=4$ and $x^2+y^2=9$, above the plane $z=0$, and below the paraboloid $z=x^2+y^2$.

2. Let E be the solid region that is inside the sphere $x^2+y^2+z^2=25$ and above the cone $z=-\sqrt{x^2+y^2}$. Assume the density is $\rho(x,y,z)=z^2$. Use a triple integral in spherical coordinates to find the mass of E.

3. Evaluate the line integral $\int_C y^2\, dx + xz\, dz$, where C is the straight line from $(1,0,2)$ to $(0,2,5)$.

4. Let $\mathbf{F}(x,y,z) = yz\mathbf{i} + (xz+2yz)\mathbf{j} + (xy+y^2)\mathbf{k}$.

   (a) Check that $\mathbf{F}$ is conservative by evaluating curl $\mathbf{F}$.

   (b) Find a function $f(x,y,z)$ such that $\mathbf{F} = \nabla f$.

5. Let E be the solid region in the first octant that is bounded by the coordinate planes and the plane $2x+y+z=6$. Let S be the surface of E, oriented outward. Let $\mathbf{F}(x,y,z) = 2x\mathbf{i}+z\mathbf{j}+y\mathbf{k}$. *Use the Divergence Theorem* to find $\int\int_S \mathbf{F}\cdot\mathbf{n}\, dS$.

6. Find the flux of $\mathbf{F}(x,y,z) = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$ across the portion of the elliptic cylinder $9x^2+4y^2=36$ that lies between the planes $z=-1$ and $z=1$. The cylinder is oriented outward. Suggestion: the cylinder can be parameterized as follows:

$$\mathbf{r}(\theta,z) = 2\cos\theta\mathbf{i} + 3\sin\theta\mathbf{j} + z\mathbf{k},\ 0\le\theta\le 2\pi,\ -1\le z\le 1.$$

I have neither given nor received unauthorized aid on this test.

(Please sign.)