MA 114-001 Test 1

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Use your own paper to work the problems. On all problems, you must **show your work** to receive credit.

When you finish, fold this paper lengthwise together with your work, so that this writing is on the outside. Write your name, row, and seat number above, and turn in.

1. In parts (a) and (b) of this problem, we give the augmented matrix in row-reduced form for a system of linear equations. For each augmented matrix, give all solutions of the system, or state that no solutions exist.

   (a)
   $$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

   (b)
   $$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

2. The inverse of the matrix
   $$\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$$
   is
   $$\begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}.$$
   Use this fact to solve the system of equations
   $$\begin{array}{rcrcr} 2x & + & 3y & = & 1 \\ 3x & + & 4y & = & -2 \end{array}$$

3. Solve the following system of linear equations by using an augmented matrix and putting it into row-reduced form.

$$\begin{array}{rcrcrcl} & & 2y & + & 6z & = & 4 \\ x & & & + & 4z & = & 2 \\ 3x & + & 2y & + & 10z & = & 2 \end{array}$$

4. Graph the region defined by the following system of inequalities. Label all corner points with their coordinates.

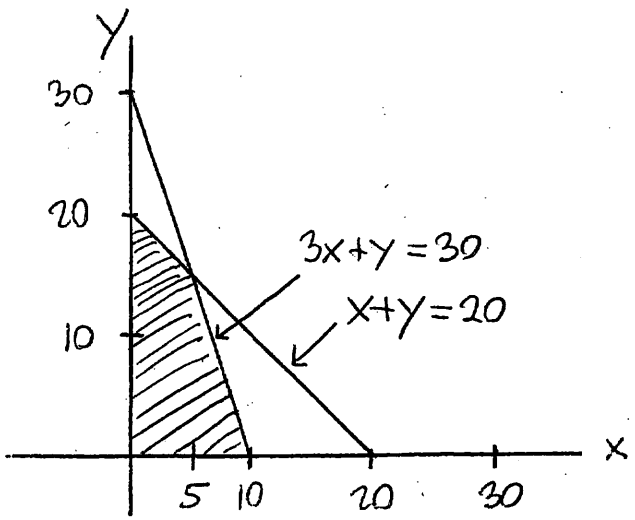
$$\begin{array}{rcrcl} x & + & 2y & \leq & 8 \\ 3x & + & y & \geq & 9 \\ & & y & \geq & 0 \end{array}$$

5. Maximize $P = 4x + y$

   subject to

$$\begin{array}{rcrcl} x & + & y & \leq & 20 \\ 3x & + & y & \leq & 30 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$

   The region defined by the inequalities is graphed below. You only need to find the maximum value of P on this region, and where that maximum occurs.



6. A lake is to be stocked with two kinds of trout, rainbows and brookies. Both these trout eat insects and crustaceans. Rainbows eat 60 insects and 10 crustaceans per day. Brookies eat 30 insects and 40 crustaceans per day. Park rangers estimate that the lake can supply 8,000 insects and 6,000 crustaceans per day. Rainbows typically grow to 5 pounds, brookies to 4 pounds. The park rangers want choose the number of rainbows and brookies to maximize the total weight of trout in the lake.

   Set up the linear programming problem that the rangers should solve. Identify the variables; state the function to be maximized or minimized; and state all the inequality constraints. **Do not graph the region or solve the problem.**