MA 114-001Final Exam

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Use your own paper to work the problems. On all problems, you must **show your work** to receive credit.

This exam is in two parts. *Please do not do Part I problems and Part II problems on the same paper.* When you finish, fold the Part I questions lengthwise together with your Part I answers, and fold the Part II questions lengthwise together with your Part II answers. Write your name, row, and seat number in the places provided, and turn in.

Part I

1. In parts (a) and (b) of this problem, we give the augmented matrix in row-reduced form for a system of linear equations. For each augmented matrix, give all solutions of the system, or state that no solutions exist.

   (a)
   $$\left(\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

   (b)
   $$\left(\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

2. Solve the following system of linear equations by using an augmented matrix and putting it into row-reduced form.

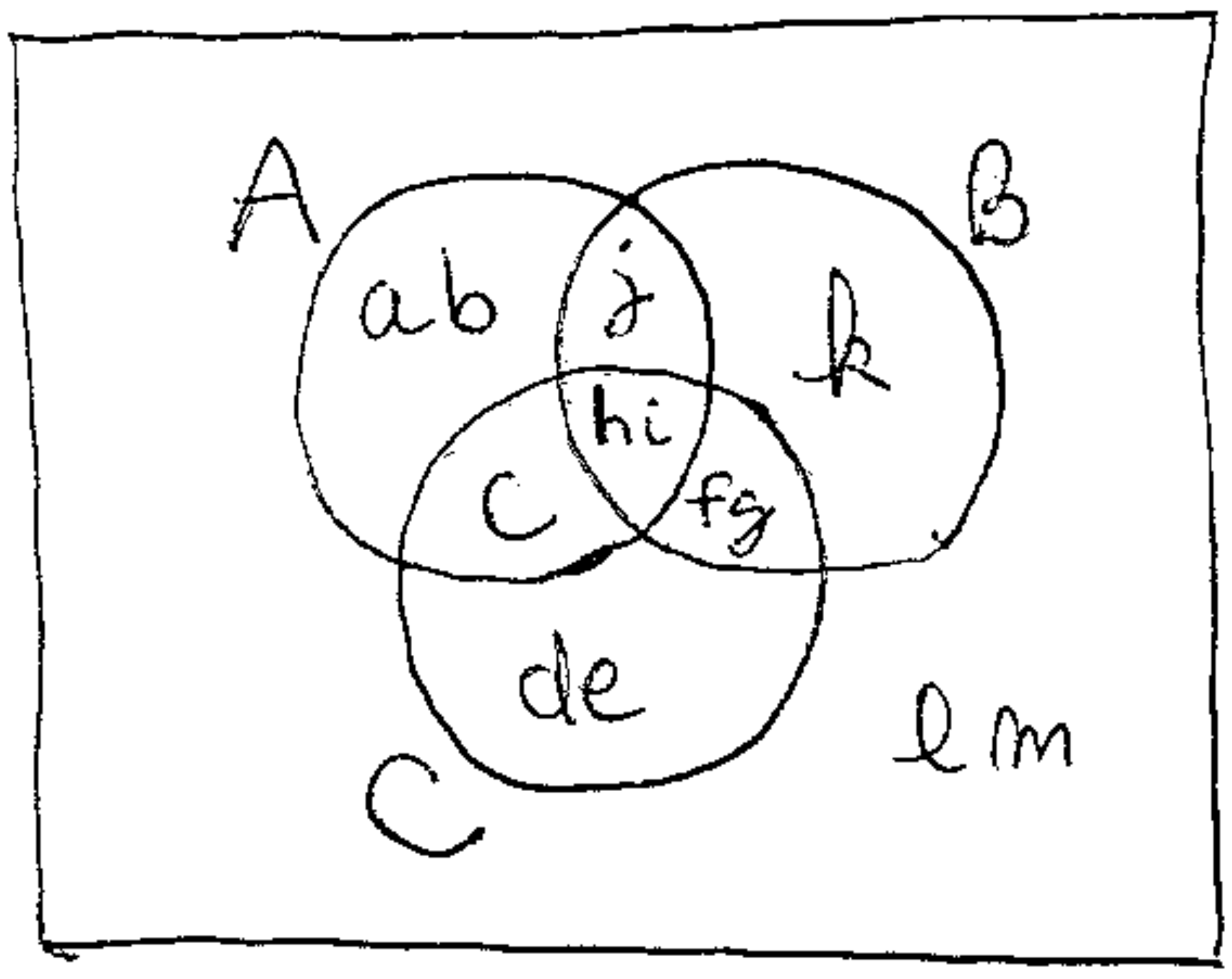
$$\begin{array}{ccccccc} x & + & 2y & & & = & 7 \\ x & + & y & + & z & = & 5 \\ 2x & & & + & z & = & 0 \end{array}$$

3. In the following Venn diagram, for which the universal set is $U = \{a, b, c, d, e, f, g, h, i, j, k, \ell, m\}$, find all the points in the following sets:

   (a) B

   (b) $(A \cup C)^c$

   (c) $A \cap B^c$



4. Your bank assigns you a four-digit personal identification number (PIN). The bank only uses the digits 1, 2, 3, 4, 5 and 6 in its PINs. Repeated digits are allowed; 1346, 1343, 3343, and 3333 are all valid PINs.

   (a) How many different PINs are there at this bank?

   (b) How many PINs are there in which all four digits are different?

   (c) If the bank randomly assigns you a PIN, what is the probability that all four digits are different?

5. John is trying to win a prize at the State Fair. He is given three coins, which he must try to toss into a jar. If he gets two coins into the jar, he wins. Hence if John gets his first two coins in the jar, he does not throw the third, since he has already won. Similarly, if John misses with his first two coins, he does not throw the third, since he has already lost.

   (a) Draw a tree diagram that shows all the possible outcomes.

   (b) Suppose that with each toss, John gets the coin in the jar with probability 1/3 and misses with probability 2/3. What is the probability that John wins the prize?

   (c) You see John walking away from the booth with a big grin and a big teddy bear, so you know he won. What is the probability that John got his first toss in the jar?

6. A coach must select a quiz bowl team of 3 students from a group of 8 girls and 4 boys who have been practicing.

   (a) How many possible teams are there if the coach does not care how many boys and how many girls are on the team?

   (b) How many possible teams are there if the coach wants to have 2 girls and 1 boy?

   (c) How many possible teams are there if the coach wants to have *at least* 2 girls?

   (d) If the coach picks the team randomly, what is the probability that he picks 2 girls and 1 boy?

Part II

1. On a typical winter night in northern Minnesota, the probability of snow is .4, the probability the electricity will fail is .3, and the probability that both will happen is .2.

   (a) What is the probability of a night with no snow and no electricity failure?

   (b) You wake up one morning in northern Minnesota, turn on the lamp, and nothing happens, so you know that the electricity failed during the night. What is the probability that it snowed?

2. An American student is trying to get a driver's license in Japan. The test has six multiple-choice questions. Each question has three possible answers, of which one is right and two are wrong. Since the student does not understand Japanese, he gives random answers to all the questions. What is the probability that the student gets exactly four questions right? (Hint: This is an independent trials process. On each question, the probability that the student gives the right answer is $1/3$, and the probability he gives the wrong answer is $2/3$.)

3. Bill's dresser drawer contains 7 blue socks and 3 white ones. He reaches into the drawer and pulls socks out one sock and then another.

   (a) Draw a tree diagram that represents this process.

   (b) What is the expected number of blue socks that Bill pulls from the drawer? (Hint: the number of blue socks that Bill pulls is either 0, 1, or 2.)

4. Every night, Nancy either goes dancing, goes to a movie, or watches television. If she goes dancing one night, the next night she either goes to a movie or watches television, each with probability $1/2$. If she goes to a movie one night, the next night she might do any of the three activities, each with probability $1/3$. If she watches television one night, she always goes dancing the next night. Set up the transition matrix for this Markov process. Label the rows and columns with letters that represent the different states.

5. A town has two radio stations, A and B. Everybody in town listens to one of the two stations. Of the people who listen to station A one month, 20% switch to station B the next month. Of the people who listen to station B one month, 10% switch to station B the next month. Hence the transition matrix for this Markov process is

$$T = \begin{pmatrix} .8 & .2 \\ .1 & .9 \end{pmatrix}.$$

Suppose that in January, 60% of the residents listen to station A and 40% listen to station B. Of the people who listen to station B.

   (a) What percentage of the residents listen to each station in February?

   (b) What percentage of the residents listen to each station in March?

   (c) In the long run, what percentage of the residents listen to each station? Answer this question by calculating the steady-state probability distribution vector.

6. A mouse is in a maze with four rooms. In room 1, a cat is hiding. If the mouse goes to room 1, the cat will eat him. In room 2, there is a door to the outside, where a piece of cheese is clearly visible. If the mouse goes to room 2, he will leave the maze, eat the cheese, and escape. Room 3 has two doors, one to room 1 and one to room 4. If the mouse is in room 3, he will leave through one of these doors, each with probability 1/2. Room 4 has three doors, one to each of the other rooms. If the mouse is in room 4, he will leave through one of these doors, each with probability 1/3. Hence the transition matrix for this Markov process is

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 & 0 \end{pmatrix}.$$

This matrix is in the canonical form

$$\begin{pmatrix} I & 0 \\ R & Q \end{pmatrix}$$

for an absorbing transition matrix.

   (a) Write down the matrix R.

   (b) Write down the matrix Q.

   (c) Calculate $N = (I - Q)^{-1}$.

   (d) Calculate NR.

   (e) Suppose the mouse starts in room 3. What is the probability that the mouse eventually escapes?

   (f) Suppose the mouse starts in room 3. What is the average number of times the mouse visits room 4 before this process ends? (Of course, it ends when the mouse is eaten or escapes.)