Game Theory and Global Warming

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What is game theory?

What is a game?

- There are several players.
- Each player has a strategy: a plan for what to do in each situation she might encounter.
- The strategies determine the outcome of the game.
- The players receive payoffs, which depend on the outcome of the game.

Example: tic-tac-toe.

Problem: Since your payoff depends not only on your own choices, but on the choices of the other players, what should you do?

Matrix games

In a two-player game, the way strategies determine payoffs can be represented by a matrix.

Example: Microsoft vs. Apple

In the early days of personal computing, you could buy a computer running Microsoft's operating system or Apple's operating system. Apple's was better, but if you needed to do a joint project with someone else, both of you needed to use the same type of computer.

		Player 2	
		M	**A**
Player 1	**M**	$(1,1)$	$(0,0)$
	A	$(0,0)$	$(2,2)$

- Rows are Player 1's strategies.
- Columns are Player 2's strategies.
- Each entry in the matrix gives first Player 1's payoff, second Player 2's payoff.

"Coordination game." Related to the present financial crisis.

First global warming game

It's time to negotiate a new treaty to stop global warming.

- Player 1: Governments of the developed world (US, Europe, Japan, ...).
- Player 2: Governments of the developing world (China, India, Brazil, Mexico, ...).

Situation:

- An investment of $2 trillion is needed to stop global warming.
- It will produce economic benefits of $4 trillion, which will be split equally between the players.
- If the investment is not made, there will be economic losses of $4 trillion, which will be split equally between the players.
- If both parties agree to contribute, each need only invest $1 trillion.
- If only one party contributes, it must invest $2 trillion.

Payoffs (benefits – investment) in trillions of dollars:

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(1,1)$	$(0,2)$
	refuse	$(2,0)$	$(-2,-2)$

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(1,1)$	$(0,2)$
	refuse	$(2,0)$	$(-2,-2)$

How to play the game:

- Governments of the developed world: blue scorecard.
- Governments of the developing world: yellow scorecard.
- Pick an opponent.
- Negotiate.
- Secretly write your move (invest or refuse).
- Read both moves.
- Record your score under ”Game 1 Total.”

Are there other payoffs?

Economic payoffs:

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(1,1)$	$(0,2)$
	refuse	$(2,0)$	$(-2,-2)$

Suppose there are additional noneconomic payoffs important to the governments of the developed world:

- If the governments of the developed world agree to invest, they receive approval from their citizens and the rest of the world that they value at \$1 trillion.
- If the governments of the developed world refuse to invest, they receive condemnation from their citizens and the rest of the world that they value at $-$\$1 trillion.

Economic plus noneconomic payoffs:

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(2,1)$	$(1,2)$
	refuse	$(1,0)$	$(-3,-2)$

Dominant strategies

One of a player's strategies *dominates* another if it gives her a better payoff against *any* choices of strategies by the other players.

First global warming game, economic payoffs only:

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(1,1)$	$(0,2)$
	refuse	$(2,0)$	$(-2,-2)$

Do any strategies dominate other strategies?

First global warming game, economic plus noneconomic payoffs:

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(2,1)$	$(1,2)$
	refuse	$(1,0)$	$(-3,-2)$

Do any strategies dominate other strategies?

Does the answer help us predict what will happen?

First global warming game, economic payoffs only:

		Developing world	
		invest	refuse
Developed world	**invest**	$(1,1)$	$(0,2)$
	refuse	$(2,0)$	$(-2,-2)$

Suppose there are additional noneconomic payoffs important to *all* governments:

- If the governments of one part of the world agree to invest, they receive approval from their citizens and the rest of the world that they value at $1 trillion.
- If the governments one part of the world refuse to invest, they receive condemnation from their citizens and the rest of the world that they value at $-$1 trillion.

Economic plus noneconomic payoffs are now:

		Developing world	
		invest	refuse
Developed world	**invest**	$(2,2)$	$(1,1)$
	refuse	$(1,1)$	$(-3,-3)$

Do any strategies dominate other strategies?

A number of organizations try to influence world's response to global warming by influencing these noneconomic payoffs.


Support Solutions
to the Climate Crisis
Join we today

Second global warming game

It's time to negotiate a new treaty to stop global warming.

- Player 1: Governments of the developed world.
- Player 2: Governments of the developing world.

Situation:

- It is proposed that the developed and developing world each invest $2 trillion to stop global warming.
- Each investment of $2 trillion will produce economic benefits of $3 trillion, which will be split equally between the players.

Payoffs (benefits – investment) in trillions of dollars:

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(1, 1)$	$(-.5, 1.5)$
	refuse	$(1.5, -.5)$	$(0, 0)$

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(1,1)$	$(-.5,1.5)$
	refuse	$(1.5,-.5)$	$(0,0)$

How to play the game:

- Governments of the developed world: yellow scorecard.
- Governments of the developing world: blue scorecard.
- Pick an opponent.
- Negotiate.
- Secretly write your move (invest or refuse).
- Read both moves.
- Record your score under ”Game 2 Total.”

Nash equilibrium

A *Nash equilibrium* in an *n*-player game is a choice of strategies, one for each player, such that no player can improve her own payoff by changing just her own strategy.

John Nash received the Nobel Prize in Economics in 1994, largely for inventing this idea. It is the most important idea in game theory.

First global warming game, economic payoffs only:

		Developing world invest	Developing world refuse
Developed world	**invest**	$(1,1)$	$(0,2)$
	refuse	$(2,0)$	$(-2,-2)$

This is a game of *Chicken*: the player who backs down from the confrontation and invests is “chicken.”

- There are two Nash equilibria. In each one player backs down, and the other gets her best possible payoff.
- If both back down, both get their second-best payoff.
- If neither backs down, both get their worst payoff.

Second global warming game:

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(1, 1)$	$(-.5, 1.5)$
	refuse	$(1.5, -.5)$	$(0, 0)$

This game is a *Prisoner's Dilemma*:

- There is a cooperative strategy and an independent strategy.
- The independent strategy dominates the cooperative strategy.
- You get your best payoff when you use the independent strategy and your opponent uses the cooperative strategy.
- In this case your opponent gets her worst payoff (sucker's payoff).
- You get your second-best payoff when you both uses the cooperative strategy.
- Unfortunately the only Nash equilibrium is to both use the independent strategy. This gives everyone their second-worst payoff.

Ideas for getting a better outcome from a Prisoner's Dilemma:

- Change the payoffs (rewards, punishments, altruistic payoffs).
- Repeat the game.

Third global warming game

Same as second global warming game:

		Developing world invest	Developing world refuse
Developed world	**invest**	$(1,1)$	$(-.5,1.5)$
	refuse	$(1.5,-.5)$	$(0,0)$

However, you will play the game twice with the same opponent.

How to play the game:

(1) Governments of the developed world: blue scorecard.
(2) Governments of the developing world: yellow scorecard.
(3) Pick an opponent.
(4) Negotiate.
(5) Secretly write your move (invest or refuse).
(6) Read both moves.
(7) Record your score under “Game 3 Score for Each Round.”
(8) Repeat (4) to (7).
(9) Record your total score under “Game 3 Total.”

Backward induction

To analyze a game with a *known, finite* number of rounds, work backward.

In the third global warming game:

(1) In round 2, it is as if one were playing a one-round game. Both players will presumably use their dominant strategy, refuse to invest.

(2) Now back up to round 1. You've already analyzed what your opponent will do in round 2. So again it is as if you were playing a one-round game. Both players will presumably use their dominant strategy, refuse to invest.

This analysis does not work if the game has an *infinite* or *unknown* number of rounds.

Fourth global warming game

Same as second global warming game:

		Developing world	
		invest	**refuse**
Developed world	**invest**	$(1, 1)$	$(-.5, 1.5)$
	refuse	$(1.5, -.5)$	$(0, 0)$

You will play a series of rounds with the same opponent. We will tell you when to stop.

How to play the game:

(1) Governments of the developed world: yellow scorecard.
(2) Governments of the developing world: blue scorecard.
(3) Pick an opponent.
(4) Negotiate.
(5) Secretly write your move (invest or refuse).
(6) Read both moves.
(7) Record your score under “Game 4 Score for Each Round.”
(8) Wait to see if we call for another round.
(9) If we do, repeat (4) to (8).
(10) If we don’t, record your total score under “Game 4 Total.”

Infinitely repeated games

An *infinitely repeated game* R consists of

- a game G that is played at times $j = 0, 1, 2, \ldots$.
- a discount factor δ, $0 < \delta < 1$.

Strategies:

- A strategy for a player in the repeated game R is just a way of choosing which of her strategies for G to use at each time.
- Her choice of which strategy to use at time j can depend on all the strategies used by all the players at times before j.

Payoffs:

- Each player gets a payoff from the game G at each time $j = 0, 1, 2, \ldots$.
- Her payoff in the repeated game R is just

$$\text{her payoff at time } 0 + \delta \cdot \text{her payoff at time } 1 + \delta^2 \cdot \text{her payoff at time } 2 + \ldots.$$

Strategies that can be used in repeated Prisoner's Dilemma games with two players:

- Grim Reaper: Start by using the cooperative strategy. Continue to cooperate as long as your opponent does. If your opponent ever chooses the independent strategy, respond by choosing the independent strategy forever.

- Tit for Tat. Start by using the cooperative strategy. After that, do whatever your opponent did in the previous round.

Can you think of others?

Theorem. In a repeated Prisoner's Dilemma game with two players, if δ is not too small, it is a Nash equilibrium for both players to use the Grim Reaper strategy. (If they do, they'll both cooperate forever.)

Idea of proof. In the global warming game you just played, for example, if both players use the Grim Reaper Strategy, each gets the payoff

$$1+\delta\cdot 1+\delta^2\cdot 1+\ldots=\frac{1}{1-\delta}$$

Suppose Player 2 tries a different strategy by, for example, refusing to invest at Round 0. Then Player 1, who invested at Round 0, will never invest again. Therefore Player 2's payoff in the repeated game will be at most

$$1.5+\delta\cdot 0+\delta^2\cdot 0+\ldots=1.5.$$

This is a lower payoff provided

$$\frac{1}{1-\delta}>1.5 \;\Leftrightarrow\; 1>1.5-1.5\delta \;\Leftrightarrow\; 1.5\delta>.5 \;\Leftrightarrow\; \delta>\frac{1}{3}.$$

Fifth global warming game

It's time to negotiate a new treaty to stop global warming.

- Six players: US, Europe, Japan, China, India, Latin America.

Situation:

- Each player must choose an amount between 0 and $1 trillion to invest to fight global warming. Allowed choices: 0, 0.1, 0.2, . . . , 0.9, 1.0.
- Total economic benefits equal twice total investments.
- Economic benefits are split equally among the six players.
- A country's payoff is its share of the benefits minus its investment.

How to play the game:

(1) Organize yourselves into groups of six: three blue scorecards, three yellow scorecards.
(2) Blue scorecards: Choose US, Europe, or Japan.
(3) Yellow scorecards: Choose China, India, or Latin America.
(4) Negotiate.
(5) Secretly write your investment (0, 0.1, 0.2, . . . , 0.9, or 1.0).
(6) Read all six moves, add them, multiply by 2, divide by 6. This is each country's benefits.
(7) Record your score (your benefits – your investment) under "Game 5 Total."

Discussion

In the game you just played, do any strategies dominate other strategies?

Are there any Nash equilibria?

Sixth global warming game

As in fifth global warming game, six players: US, Europe, Japan, China, India, Latin America.

- Each player must choose an amount between 0 and $1 trillion to invest to fight global warming. Allowed choices: 0, 0.1, 0.2, . . . , 0.9, 1.0.
- Total economic benefits equal twice total investments.
- Economic benefits are split equally among the six players.
- A country's payoff is its share of the benefits minus its investment.

How to play the game:

(1) Organize yourselves into groups of six: three yellow scorecards, three blue scorecards.
(2) Yellow scorecards: Choose US, Europe, or Japan.
(3) Blue scorecards: Choose China, India, or Latin America.
(4) Negotiate.
(5) Secretly write your investment (0, 0.1, 0.2, . . . , 0.9, or 1.0).
(6) Read all moves, add them, multiply by 2, divide by 6. This is each country's benefits.
(7) Record your score under "Game 6 Score for Each Round."
(8) Wait to see if we call for another round.
(9) If we do, repeat (4) to (7).
(10) If we don't, record your total score under "Game 6 Total."

Concluding comments

Nash equilibria

- A benefit of looking for Nash equilibria in a game: prevents wishful thinking.
- Nash equilibria can be where the players “get stuck.”

What is game theory good for?

- Clarify thinking.
- Explain the world. (Find Nash equilibria, one of them may be what you are seeing.)
- Respond to the world. (Pick your strategy.)
- Change the world.

Two thoughts

- Modeling reality is hard. Modeling what we believe is easier.
- Game theory converts second-order effects into first-order effects.