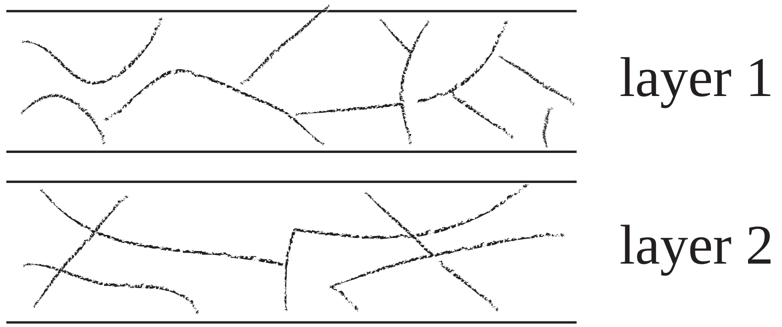
Combustion fronts in porous media with two layers



Steve Schecter

North Carolina State University (USA)

Jesus Carlos da Mota

Universidade Federal de Goiás (Brazil)

Overview

Subject: Propagation of a combustion front through a porous medium with two parallel layers having different properties.

- Each layer admits a traveling combustion wave.
- The layers are coupled by heat transfer.

Question: Does the two-layer system admit a traveling combustion wave?

Answer, for a very simplified model:

- Small heat transfer: Yes, if the combustion waves in individual layers have approximately equal speed.
- Large heat transfer: Yes.

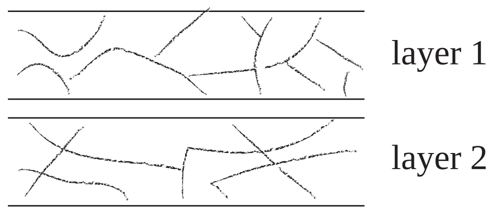
Motivations:

- Relevant to "thermally enhanced oil recovery."
- Little work on traveling waves in coupled systems. Exception: Amitabha Bose, *Symmetric and antisymmetric pulses in parallel coupled nerve fibres*, SIAM J. Appl. Math., vol. 55 (1995), pp. 1650–1674.

Thanks to: Dan Marchesin (IMPA, Rio de Janeiro).

Model

Porous medium with two layers:



In each layer:

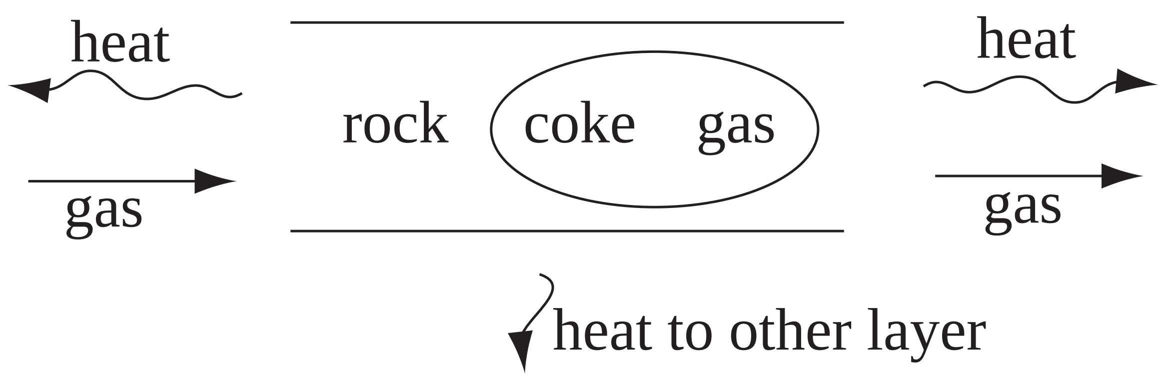
- porous rock
- coke (fuel)
- gas = oxygen + other

We ignore heat loss to surrounding rock.

Reaction: [coke] + [oxygen] $\rightarrow$ [other gas] + [heat]

Balance of energy in each layer:

$$\frac{\partial}{\partial t}\,[\text{rock energy} + \text{gas energy} + \text{coke energy}] =$$
$$-\frac{\partial}{\partial x}\,[\text{flux of gas energy}] + [\text{heat produced by reaction}]$$
$$-\,[\text{heat transported to other layer}] + [\text{diffusion of heat}]$$



Balance of coke mass in each layer:

$$\frac{\partial}{\partial t}\,[\text{coke mass}] = -\,[\text{rate of consumption of coke in reaction}]$$

- Assume the gas is incompressible, with specific volume and pressure independent of its composition.
- Then "seepage velocity" of gas is constant, and we can ignore balance of mass of gas and oxygen.

Dimensionless variables:

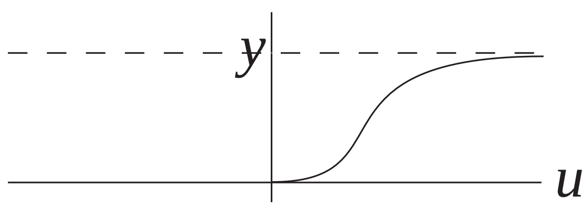
- $u(x,t)$ temperature of layer 1
- $y(x,t)$ remaining coke in layer 1
- $v(x,t)$ temperature of layer 2
- $z(x,t)$ remaining coke in layer 2

Normalizations:

- "Reservoir temperature" is $u = v = 0$: no reaction.
- $0 \leq y,\ z \leq 1$

Arrhenius reaction-rate function:

$$f(u,y) = \begin{cases} ye^{-\frac{E}{u}} & \text{if } u > 0, \\ 0 & \text{if } u \leq 0. \end{cases}$$



Graph of f for fixed y

PDEs for two-layer model:

$$\frac{\partial}{\partial t}\Big((a_1 + b_1 y)u\Big) + \frac{\partial}{\partial x}\Big(c_1 u\Big) = d_1 f(u, y) - q(u - v) + \lambda_1 \frac{\partial^2 u}{\partial x^2}\,, \tag{0.1}$$

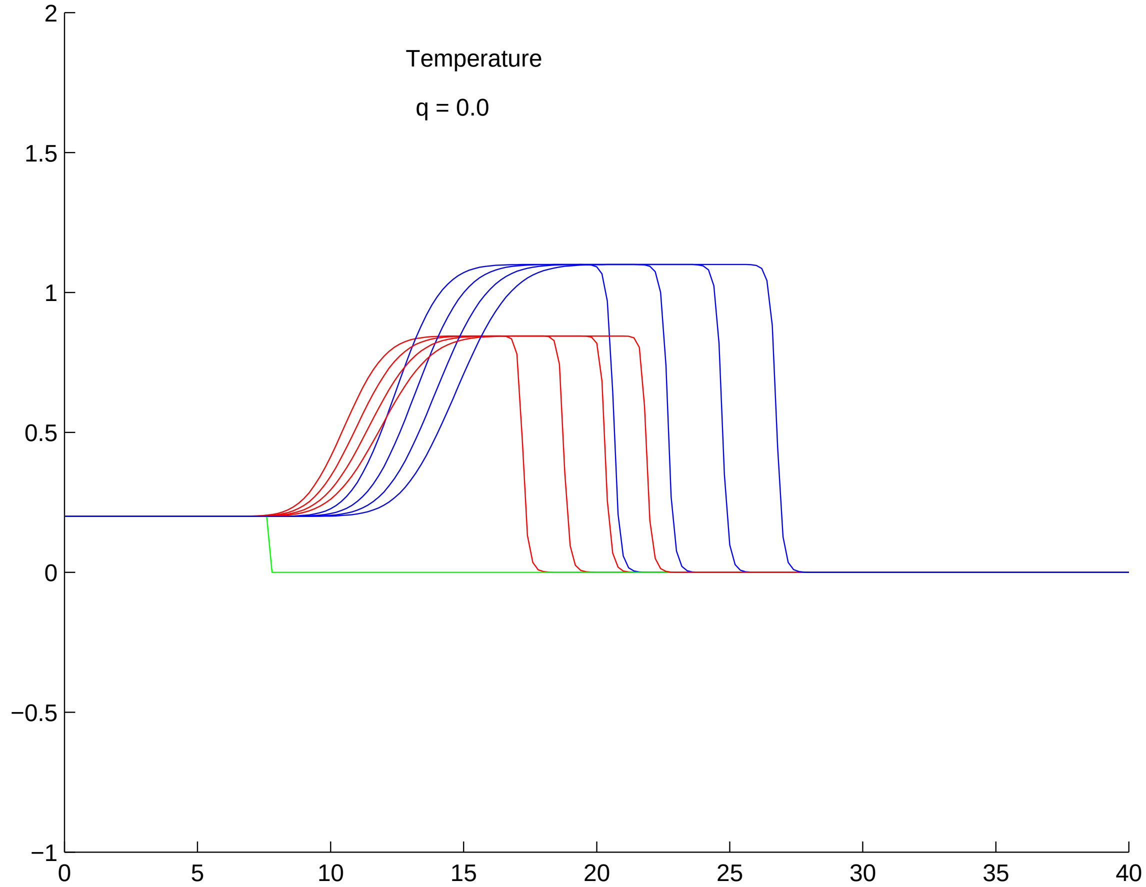
$$\frac{\partial y}{\partial t} = -A_1 f(u, y)\,, \tag{0.2}$$

$$\frac{\partial}{\partial t}\Big((a_2 + b_2 z)v\Big) + \frac{\partial}{\partial x}\Big(c_2 v\Big) = d_2 f(v, z) - q(v - u) + \lambda_2 \frac{\partial^2 v}{\partial x^2}\,, \tag{0.3}$$

$$\frac{\partial z}{\partial t} = -A_2 f(v, z)\,. \tag{0.4}$$

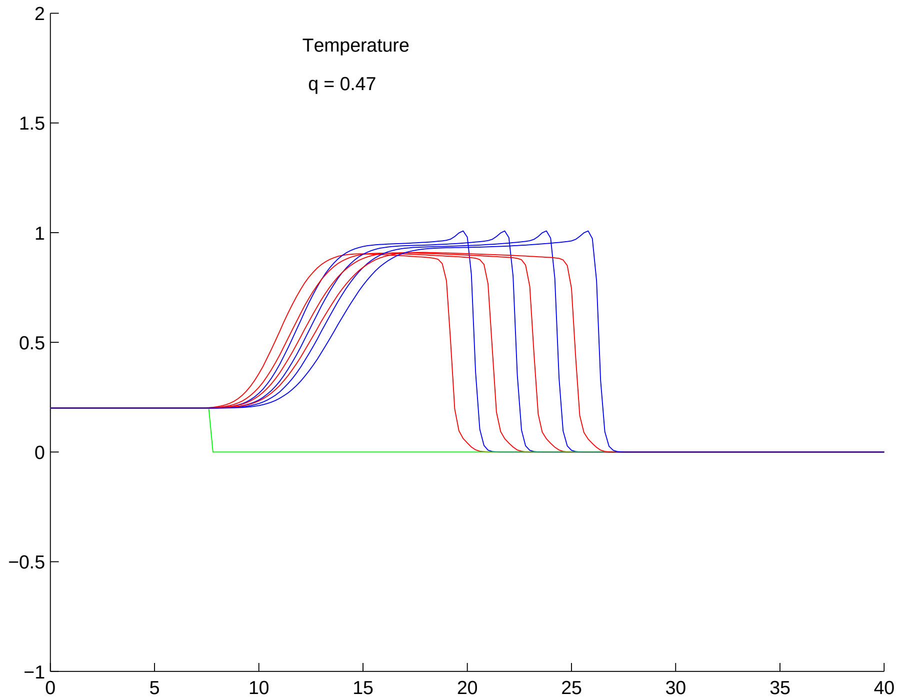
Finite-interval simulations

Simulation with $q = 0$:



In each layer a traveling wave forms, but the waves have different left states (combustion temperature) and different speeds.

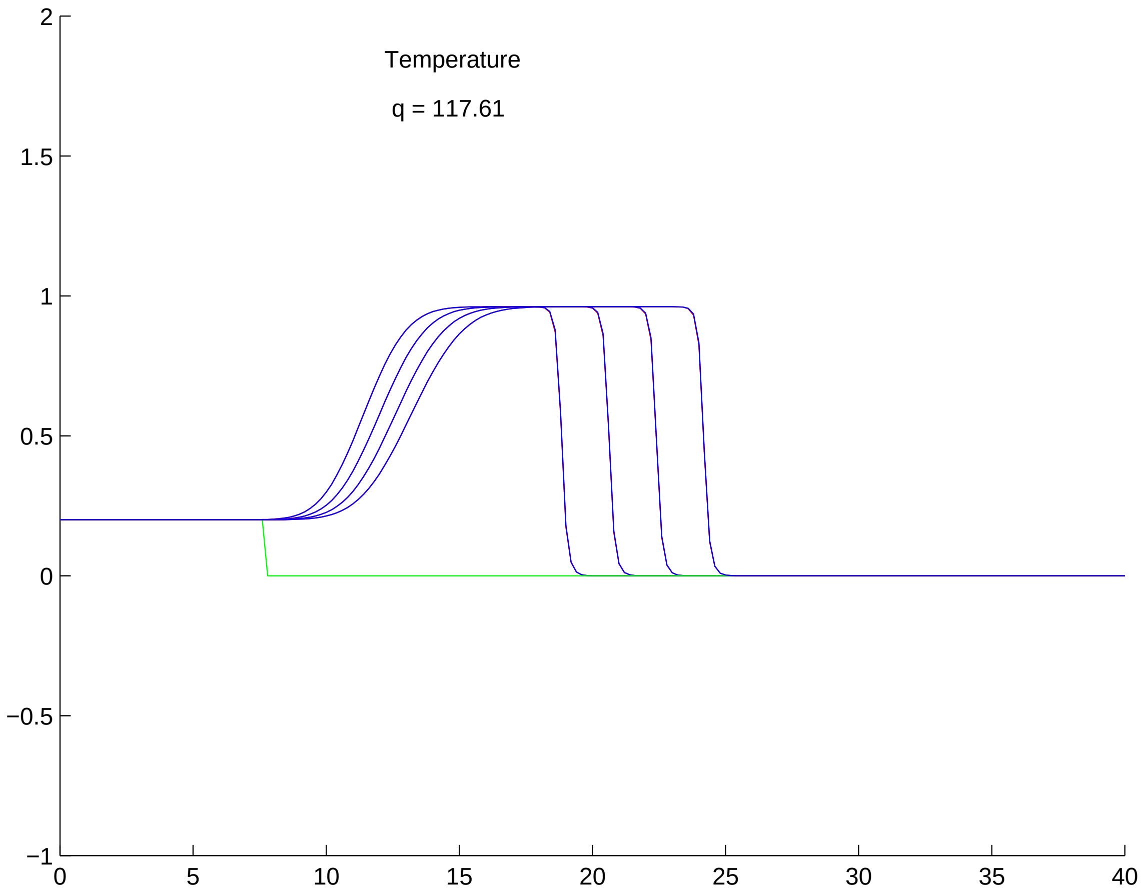
Simulation with $q = 0.47$:



Moving from right to left:

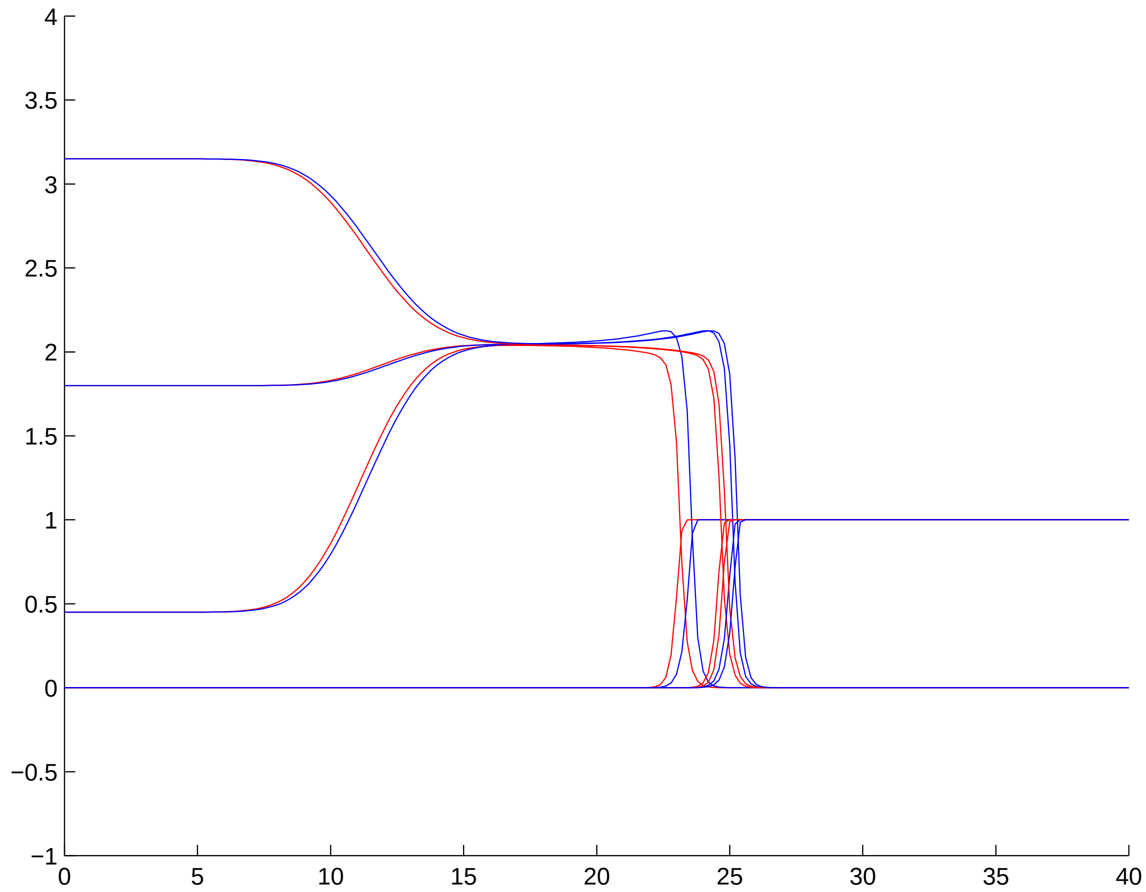
- Ahead of the combustion front the temperature is 0 (the initial temperature).
- In the combustion zones, which differ in the two layers, the temperature rises rapidly to the layer combustion temperature, which is higher in layer 1 than layer 2.
- Behind the combustion zones the temperatures equilibrate at an intermediate value. The part of the solution from here to the right propagates as a traveling wave.
- Behind this point the solution in each layer approaches the boundary condition.

Simulation with $q = 117.61$:



Temperature profiles in the two layers are extremely close.

Simulations with different left boundary and initial conditions:



The traveling combustion wave that forms is independent of the injection temperature. It depends only on the system parameters and q.

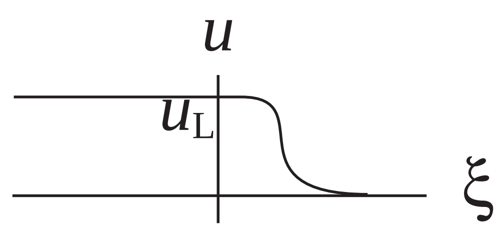
For all simulations we use typical parameter values from the engineering literature:

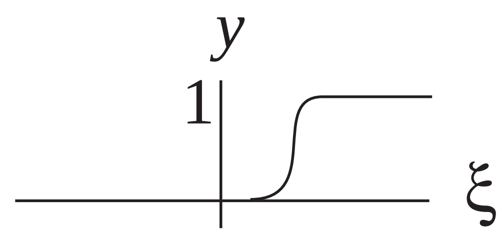
$$
\begin{aligned}
a_1 &= 19.4840, \\
a_2 &= 16.7032, \\
b_1 &= 1.0, \\
b_2 &= 1.0, \\
c_1 &= 1.8054, \\
c_2 &= 2.1063, \\
d_1 &= 62.5415, \\
d_2 &= 50.9796, \\
A_1 &= 5.4093, \\
A_2 &= 4.4093, \\
\lambda_1 &= 0.0815, \\
\lambda_2 &= 0.0815.
\end{aligned}
$$

We shift and rescale the temperature so that 0 corresponds to the initial temperature of the porous medium T_0, and 1 corresponds to $T_0 + 773.15$.

Traveling wave system

Look for traveling wave solution $\big(u(\xi), y(\xi), v(\xi), z(\xi)\big)$, $\xi = x - \sigma t$, satisfying the boundary conditions





- At the left, temperature is high, all coke is burned.
- At the right, "reservoir temperature," all coke is present.
- u_L, v_L, and σ are unknown.

$$(u, y, v, z)(-\infty) = (u_L, 0, v_L, 0) \quad \text{(burned)},$$

$$(u, y, v, z)(\infty) = (0, 1, 0, 1) \quad \text{(unburned)},$$

$$(\dot{u}, \dot{y}, \dot{v}, \dot{z})(-\infty) = (\dot{u}, \dot{y}, \dot{v}, \dot{z})(\infty) = (0, 0, 0, 0).$$

Second-order traveling wave system:

$$-\sigma\frac{d}{d\xi}\big((a_1+b_1y)u\big)+\frac{d}{d\xi}(c_1u)=d_1f(u,y)-q(u-v)+\lambda_1\frac{d^2u}{d\xi^2}\,, \tag{0.5}$$

$$\sigma\frac{dy}{d\xi}=A_1f(u,y)\,, \tag{0.6}$$

$$-\sigma\frac{d}{d\xi}\big((a_2+b_2z)v\big)+\frac{d}{d\xi}(d_2v)=d_2f(v,z)+q(u-v)+\lambda_2\frac{d^2v}{d\xi^2}\,, \tag{0.7}$$

$$\sigma\frac{dz}{d\xi}=A_2f(v,z)\,. \tag{0.8}$$

To convert to a first-order system:

Integrate (0.5)–(0.8) from ξ to $+\infty$, and use $(u,y,v,z)(\infty) = (0,1,0,1)$ and $\dot{u}(\infty)=\dot{v}(\infty)=0$:

$$\lambda_1 \dot{u} = -\sigma(a_1 + b_1 y)u + c_1 u + d_1 \int_\xi^\infty f(u,y)\, d\tilde{\xi} - \int_\xi^\infty q(u-v)\, d\tilde{\xi}\,,$$

$$\sigma(1-y) = A_1 \int_\xi^\infty f(u,y)\, d\tilde{\xi}\,,$$

$$\lambda_2 \dot{v} = -\sigma(a_2 + b_2 z)v + c_2 v + d_2 \int_\xi^\infty f(v,z)\, d\tilde{\xi} - \int_\xi^\infty q(v-u)\, d\tilde{\xi}\,,$$

$$\sigma(1-z) = A_2 \int_\xi^\infty f(v,z)\, d\tilde{\xi}\,.$$

Let

$$w(\xi) = \int_\xi^\infty q(u-v)\, d\tilde{\xi},$$

the total heat difference between the layers from ξ to the right.

Two-layer traveling-wave system:

$$\dot{u} = \frac{1}{\lambda_1}\left(-\sigma\big(b_1 u y + a_1 u + \frac{d_1}{A_1}(y-1)\big) + c_1 u - w\right), \tag{0.9}$$

$$\dot{y} = \frac{1}{\sigma} A_1 f(u, y), \tag{0.10}$$

$$\dot{v} = \frac{1}{\lambda_2}\left(-\sigma\big(b_2 v z + a_2 v + \frac{d_2}{A_2}(z-1)\big) + c_2 v + w\right), \tag{0.11}$$

$$\dot{z} = \frac{1}{\sigma} A_2 f(v, z), \tag{0.12}$$

$$\dot{w} = q(v - u). \tag{0.13}$$

Boundary conditions:

$$(u, y, v, z, w)(-\infty) = (u_L, 0, v_L, 0, w_L) \quad \text{(burned)},$$
$$(u, y, v, z, w)(\infty) = (0, 1, 0, 1, 0) \quad \text{(unburned)}.$$

One-layer traveling wave system: Set $w = 0$ in (0.9)–(0.10), with boundary conditions

$$(u, y)(-\infty) = (u_L, 0), \quad (u, y)(\infty) = (0, 1).$$

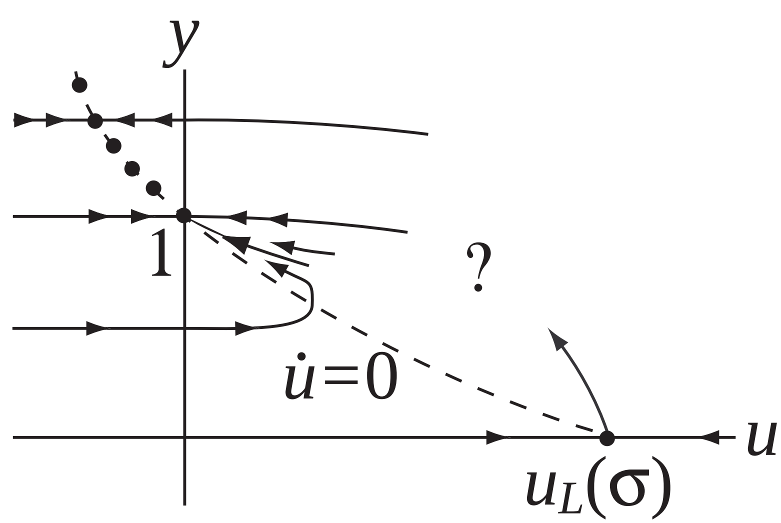
Combustion fronts in one layer

One-layer traveling wave system (system parameters fixed):

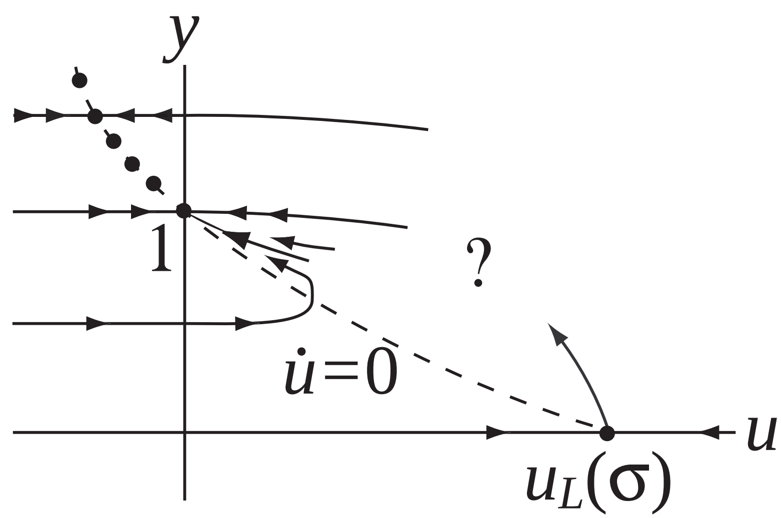
$$\dot{u} = \frac{1}{\lambda_1}\left(-\sigma\big(b_1 uy + a_1 u + \frac{d_1}{A_1}(y-1)\big) + c_1 u\right), \tag{0.14}$$

$$\dot{y} = \frac{1}{\sigma} A_1 f(u, y). \tag{0.15}$$

Phase portrait for $\sigma > \frac{c_1}{a_1}$:



We want a connecting orbit from the hyperbolic saddle $U_L(\sigma) = (u_L(\sigma), 0)$ to the nonhyperbolic equilibrium $U_R = (0, 1)$.



Definition 0.1. A connecting orbit from a hyperbolic equilibrium U_L to a non-hyperbolic equilibrium U_R is called *strong* if it lies in the stable manifold of U_R. The corresponding traveling wave is also called strong.

Theorem 0.2. *There is a number σ^*, with $\frac{c_1}{a_1} < \sigma^* < \infty$, such that the one-layer system has a connecting orbit from $U_L(\sigma)$ to U_R if and only if $\sigma * \leq \sigma < \infty$. The connecting orbit is strong if and only if $\sigma = \sigma^*$.*

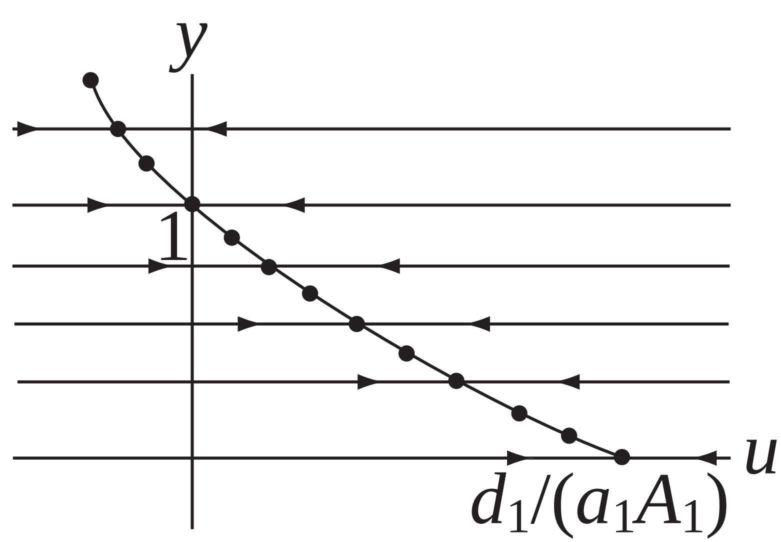
Proof. 1. Phase portrait of (0.14)–(0.15) for large σ:

Let $\sigma = \frac{1}{\epsilon}$, multiply the resulting system by by ϵ:

$$\dot{u} = \frac{1}{\lambda_1}\left(-\left(b_1 uy + a_1 u + \frac{d_1}{A_1}(y-1)\right) + \epsilon c_1 u\right), \tag{0.16}$$

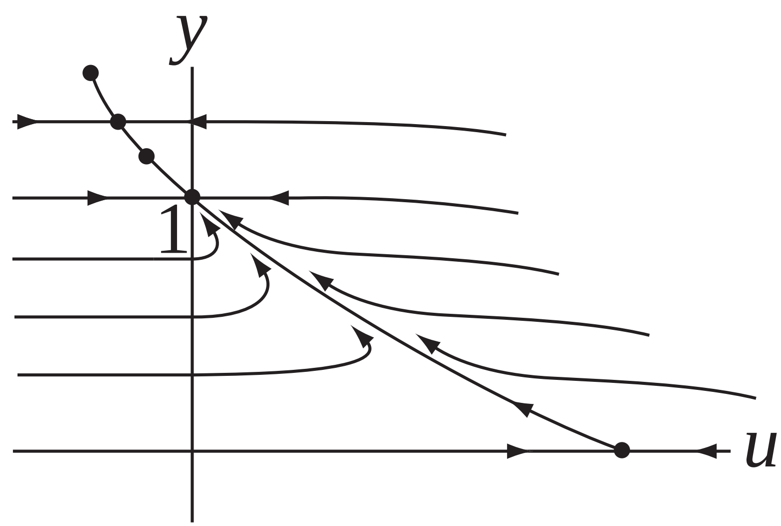
$$\dot{y} = \epsilon^2 A_1 f(u,y). \tag{0.17}$$

"Fast-slow" form. Phase portrait for $\epsilon = 0$:



- There is a normally hyperbolic curve of equilibria.
- Therefore for small ϵ, near it there is a normally hyperbolic invariant curve.

Phase portrait for small ϵ or large σ:



- For large σ, there are connections, but they are not strong.

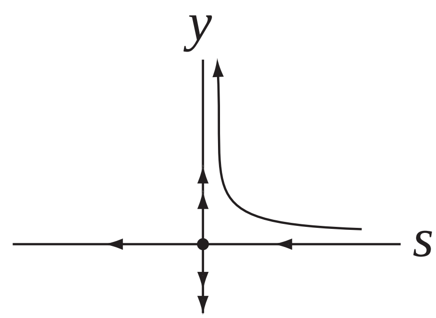
2. Phase portrait of (0.14)–(0.15) for small $\sigma > \frac{c_1}{a_1}$:

Let $\sigma = \frac{c_1}{a_1} + \tau$. Since $u(\sigma) \to \infty$ as $\tau \to 0$, also let $u = \frac{1}{s}$. We obtain the following system for $s > 0$ (corresponding to $u > 0$):

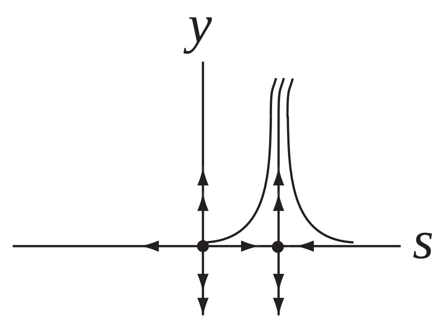
$$\dot{s} = \frac{s}{\lambda_1}\left((\frac{c_1}{a_1} + \tau)\big(b_1 y + a_1 + \frac{d_1}{A_1}(y-1)s\big) - c_1 \right), \tag{0.18}$$

$$\dot{y} = \frac{A_1 a_1}{c_1 + a_1 \tau} y e^{-Es}. \tag{0.19}$$

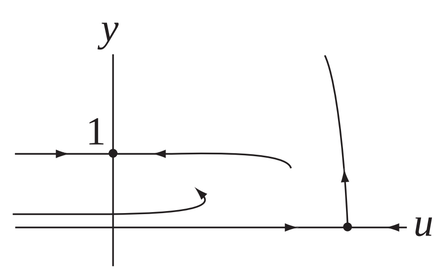
Phase portrait for $\tau = 0$:



Phase portrait for small $\tau > 0$:

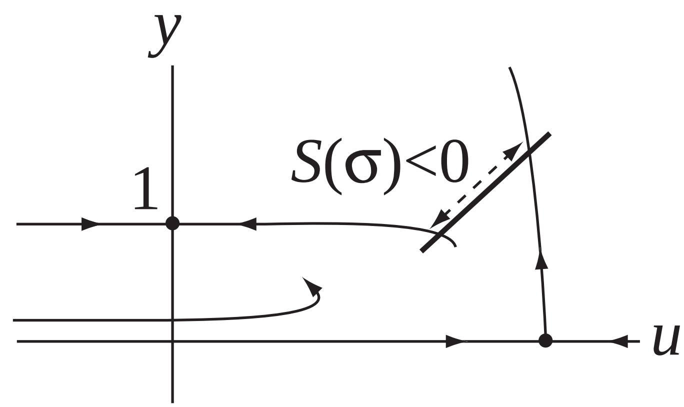


Phase portrait of (0.14)–(0.15) for small $\sigma > \frac{c_1}{a_1}$:



- For small $\sigma > \frac{c_1}{a_1}$, there is no connection.
- There must be a connection for some intermediate σ^*.

3. Define a separation function $S(\sigma)$ between $W^u(U_L(\sigma))$ and $W^s(U_R)$.



At a value $\sigma = \sigma^*$ where the separation is 0 (i.e., where there is a strong connection), let the connecting solution be $(u^*(t), y^*(t))$, and let

$$T(t) = \frac{\partial \dot{u}}{\partial u}(u^*(t), y^*(t)) + \frac{\partial \dot{y}}{\partial y}(u^*(t), y^*(t)).$$

Then we have the well-known Melnikov integral formula

$$S'(\sigma^*) = \int_{-\infty}^{\infty} \exp\left(-\int_0^t T(s)\, ds\right) (\dot{u}^*(t), \dot{y}^*(t)) \cdot \left(\frac{\partial \dot{y}}{\partial \sigma}(u^*(t), y^*(t)), -\frac{\partial \dot{u}}{\partial \sigma}(u^*(t), y^*(t))\right) dt.$$

This is positive.

- Therefore the value of σ at which a strong connection exists is unique.

□

We regard only strong traveling waves as physical.

Reasons:

- In numerical simulations of the PDE, only strong combustion fronts seem to form.

- If $(u^*(\xi), y^*(\xi))$ is a strong connection, $\int_{\xi_0}^{\infty} u^*(\xi)\, d\xi$ is finite, because $u(\xi)$ approaches 0 exponentially. Thus the total heat ahead of the combustion front is finite. This conclusion does not hold for connections that are not strong. Since the source of the heat is the combustion front, how could there be an infinite amount of heat ahead of it?

- In other combustion contexts, the addition to the equations of perturbation terms such as heat loss to the rock formation causes the zero eigenvalues at unburned states to become positive, thus eliminating connections that are not strong. (However, the unburned states remain equilibria, even though they are nonhyperbolic before perturbation.)

Two-layer traveling wave system: equilibria

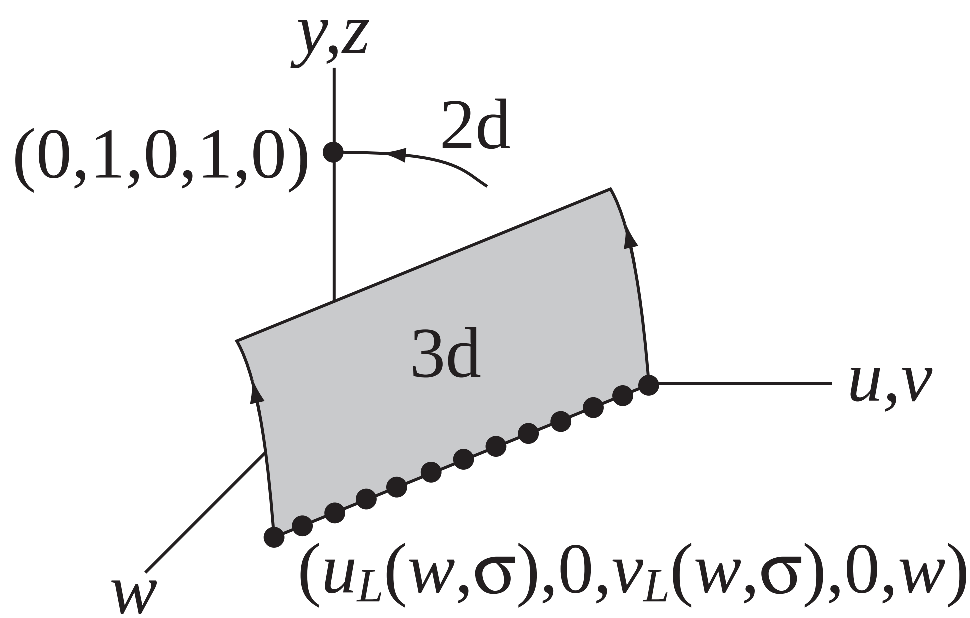
$$
\begin{aligned}
\dot{u} &= \frac{1}{\lambda_1}\left(-\sigma\big(b_1 u y + a_1 u + \frac{d_1}{A_1}(y-1)\big) + c_1 u - w\right),\\
\dot{y} &= \frac{1}{\sigma} A_1 f(u,y),\\
\dot{v} &= \frac{1}{\lambda_2}\left(-\sigma\big(b_2 v z + a_2 v + \frac{d_2}{A_2}(z-1)\big) + c_2 v + w\right),\\
\dot{z} &= \frac{1}{\sigma} A_2 f(v,z),\\
\dot{w} &= q(v-u).
\end{aligned}
$$

Restrict to: $0 \le y,\ z \le 1$.

Assume: $\sigma > \max\left(\frac{c_1}{a_1}, \frac{c_2}{a_2}\right)$.

For $q = 0$:

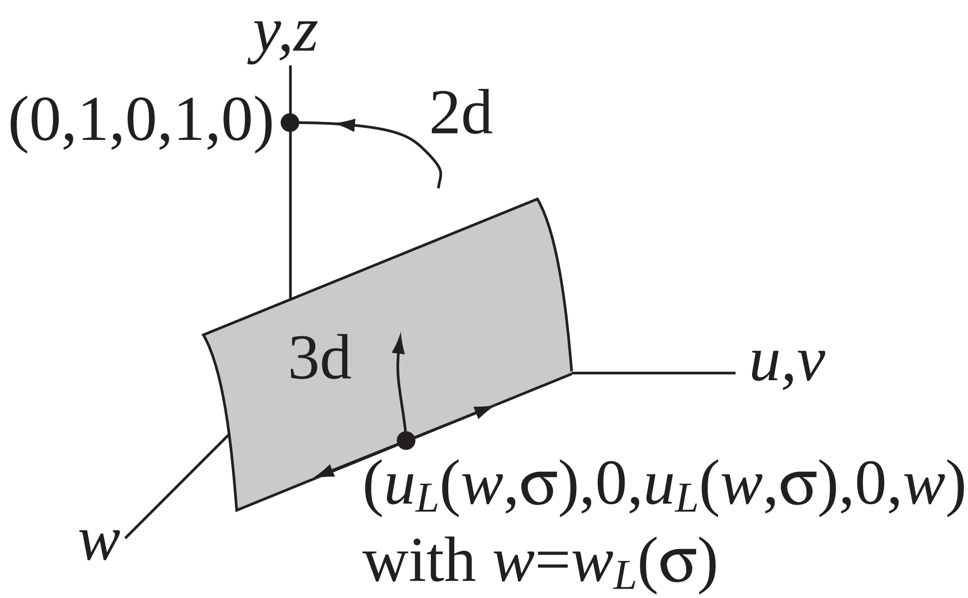
- $\dot{w} = 0$.
- There is a normally hyperbolic curve of equilibria with $y = z = 0$.
- There is a nonhyperbolic equilibrium $X_R = (0,1,0,1,0)$.



There is one value of w, $w = w_L(\sigma)$, for which $u_L(\sigma) = v_L(\sigma)$.

For $q > 0$:

- There is a normally hyperbolic invariant curve of equilibria with $y = z = 0$.
- It contains one equilibrium $X_L(\sigma)$, for which $u = v$ and $w = w_L(\sigma)$.
- There is a nonhyperbolic equilibrium $X_R = (0, 1, 0, 1, 0)$.



We want a strong connecting orbit from $X_L(\sigma)$ to X_R.

We expect strong connections for isolated values of σ.

Combustion fronts in two layers for large q

Fix the system parameters.

Theorem 0.3. *If q is sufficiently large, there is a wave speed $\sigma^*(q)$ for which the traveling wave system has a strong connection from $X_L(\sigma^*)$ to X_R.*

Proof. Let

$$q = \frac{1}{\epsilon^2}, \quad w = \frac{\omega}{\epsilon}, \quad \xi = \epsilon\eta.$$

Using $'$ to denote $\frac{d}{d\eta}$, we obtain

$$u' = \epsilon h(u, y, \sigma) - \frac{\omega}{\lambda_1}, \tag{0.20}$$

$$v' = \epsilon k(v, z, \sigma) + \frac{\omega}{\lambda_2}, \tag{0.21}$$

$$y' = \epsilon \frac{A_1}{\sigma} f(u, y), \tag{0.22}$$

$$z' = \epsilon \frac{A_2}{\sigma} f(v, z), \tag{0.23}$$

$$\omega' = v - u, \tag{0.24}$$

Set $\epsilon = 0$:

$$u' = -\frac{\omega}{\lambda_1}\,, \tag{0.25}$$

$$v' = \frac{\omega}{\lambda_2}\,, \tag{0.26}$$

$$y' = 0\,, \tag{0.27}$$

$$z' = 0\,, \tag{0.28}$$

$$\omega' = v - u\,. \tag{0.29}$$

A linear equation. There is a 3-dimensional plane of equilibria

$$S_0 = \{(u, v, y, z, \omega) : v = u \text{ and } \omega = 0\}.$$

The eigenvalues of (0.25)–(0.29) are 0, 0, 0, $\pm(\frac{1}{\lambda_1} + \frac{1}{\lambda_2})^{\frac{1}{2}}$, so S_0 is a normally hyperbolic.

For small $\epsilon > 0$, any compact part of S_0 perturbs to a normally hyperbolic invariant manifold S_ϵ. In S_ϵ, $v \approx u$ everywhere.

For variables on S_ϵ we use (u, y, z).

DE on S_ϵ, in the variables (u, y, z), in the slow time ξ:

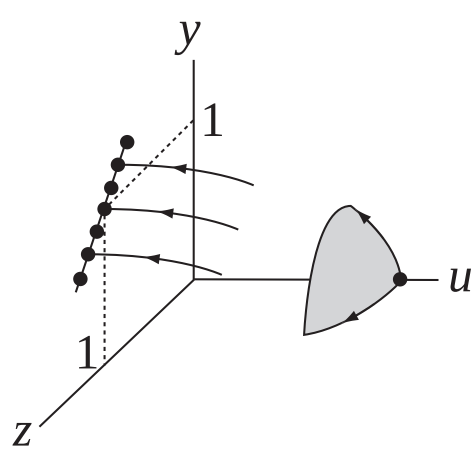
$$\dot{u} = \frac{1}{\lambda_1 + \lambda_2}(\lambda_1 h(u, y, \sigma) + \lambda_2 k(u, y, \sigma)) \ , \tag{0.30}$$

$$\dot{y} = \frac{A_1}{\sigma} f(u, y) \, , \tag{0.31}$$

$$\dot{z} = \frac{A_2}{\sigma} f(u, z) \, , \tag{0.32}$$

plus terms of order ϵ. With $\epsilon = 0$, this describes one-layer with two fuels.

Phase portrait on S_ϵ to lowest order for $\sigma > \frac{c_1+c_2}{a_1+a_2}$;



Study large σ and small $\sigma > \frac{c_1+c_2}{a_1+a_2}$ as in the one-layer case. □

We have not been able to show σ^* is unique.

Combustion fronts in two layers for small q

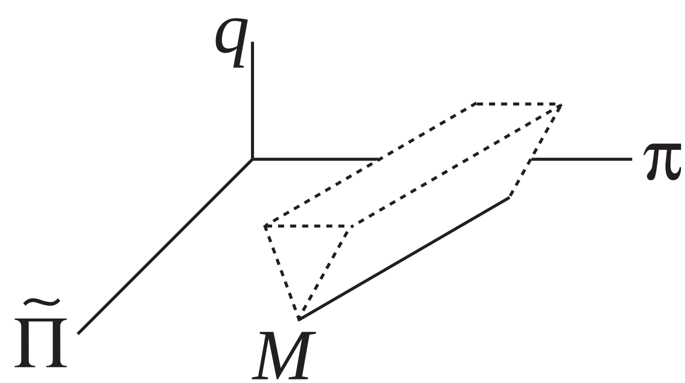
P = system parameter space.

For $\Pi \in P$ and $q = 0$, there is a strong traveling wave in layer i with speed $\sigma_i(\Pi)$.

$M = \{\Pi \in P : \sigma_1(\Pi) = \sigma_2(\Pi)\}$.

A point $\Pi \in M$ is *regular* if $D(\sigma_1 - \sigma_2)(\Pi) \neq 0$.

Theorem 0.4. *Let Π^* be a regular point of M, and let $\sigma^* = \sigma_1(\Pi^*) = \sigma_2(\Pi^*)$. Then for small $q > 0$ there is an open subset U_q of P near Π^* where there is a strong traveling wave with speed near σ^*.*



Explanation:

Write $\Pi = (\tilde{\Pi}, \pi)$, where π is chosen so that at $\Pi^* = (\tilde{\Pi}^*, \pi^*)$, the partial derivative $\sigma_1 - \sigma_2$ in the π-direction is nonzero.

Fix $\tilde{\Pi} = \tilde{\Pi}^*$.

Write the traveling wave system as

$$\dot{U} = G(U, w, \sigma, \pi), \tag{0.33}$$

$$\dot{V} = H(V, w, \sigma, \pi), \tag{0.34}$$

$$\dot{w} = q(v - u), \tag{0.35}$$

$$\dot{\sigma} = 0, \tag{0.36}$$

$$\dot{\pi} = 0, \tag{0.37}$$

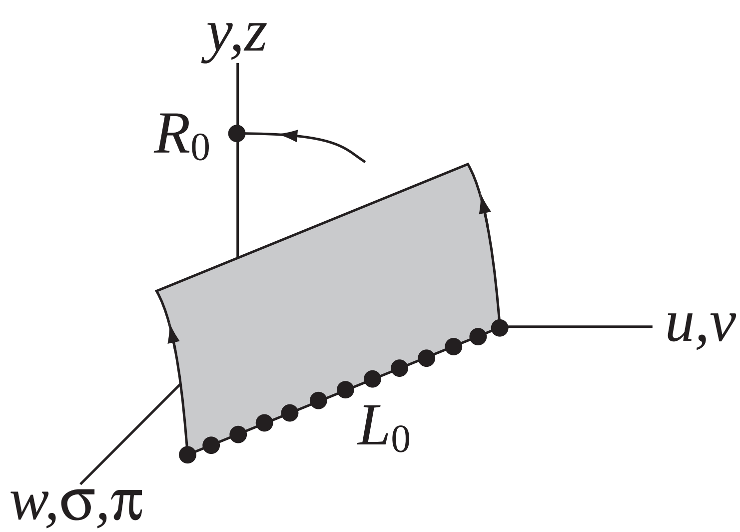
with $U = (u, y)$, $V = (v, z)$, $G = (G_1, G_2)$ and $H = (H_1, H_2)$.

State space is now $UVw\sigma\pi$-space (dimension $= 7$), and q is a parameter.

In short: $\dot{X} = F(X, q)$, $X = (U, V, w, \sigma, \pi)$.

Important equilibria for $q = 0$:

$$L_0 = \{(u, y, v, z, w, \sigma, \pi) : u = u_L(w, \sigma, \pi) = \frac{\sigma d_1 - A_1 w}{A_1(\sigma a_1 - c_1)},$$
$$v = v_L(w, \sigma, \pi) = \frac{\sigma d_2 + A_2 w}{A_2(\sigma a_2 - c_2)},\ y = z = 0\},$$
$$R_0 = \{(u, y, v, z, w, \sigma, \pi) : u = v = w = 0 \text{ and } y = z = 1\}.$$



- L_0 has dimension 3, it's a normally hyperbolic manifold of equilibria. $W^u(L_0)$ has dimension 5
- R_0 has dimension 2. $W^s(R_0)$ has dimension 4

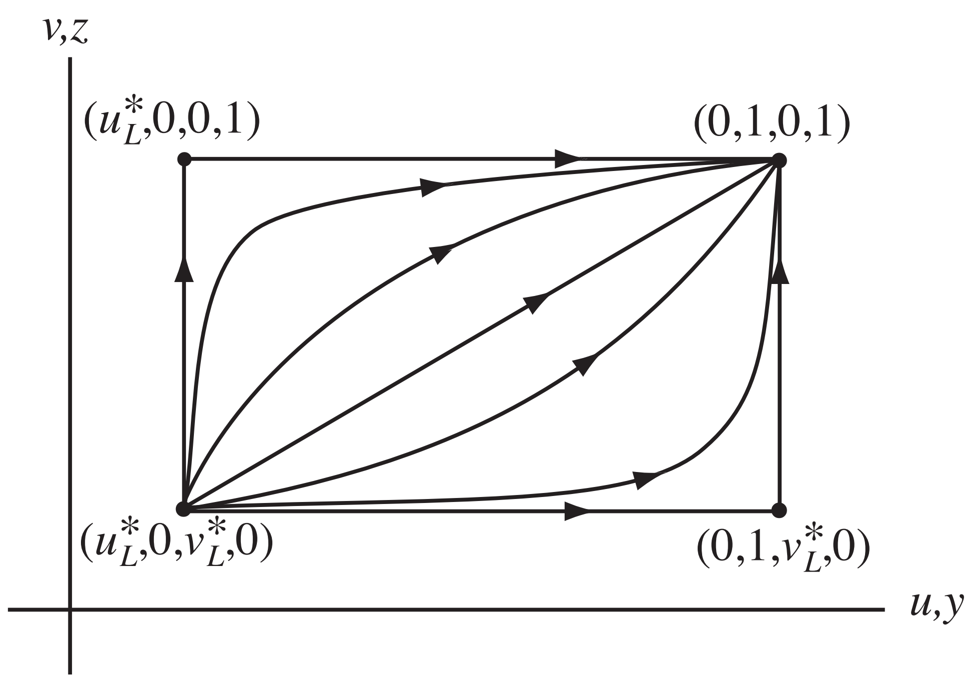
We expect $W^u(L_0) \cap W^s(R_0)$ to have dimension 2. What is it?

Let

- $u_L^* = u_L(0, \sigma^*, \pi^*)$.
- $v_L^* = v_L(0, \sigma^*, \pi^*)$.

$W^u(L_0)$ and $W^s(R_0)$ meet transversally along the a two-dimensional sheet of connections from $(u_L^*, 0, v_L^*, 0, 0, \sigma^*, \pi^*)$ to $(0, 1, 0, 1, 0, \sigma^*, \pi^*)$:

$$(U, V, w, \sigma, \pi) = (U^*(\xi), V^*(\xi + \eta), 0, \sigma^*, \pi^*).$$



Same picture is in paper of Bose.

For small $q > 0$:

- R_0 persists as a 2-dimensional set of equilibria R_q.
- $W^s(R_q)$ has dimension 4.
- There is a 3-dimensional normally hyperbolic invariant manifold L_q near L_0.
- $W^u(L_q)$ has dimension 5.
- $W^u(L_q) \cap W^s(R_q)$ has dimension 2 (persistence of transversal intersections).

What do orbits in $W^u(L_q) \cap W^s(R_q)$ look like?

DE on L_q to lowest order, in slow time:

$$w' = v_L(w, \sigma, \pi) - u_L(w, \sigma, \pi), \tag{0.38}$$

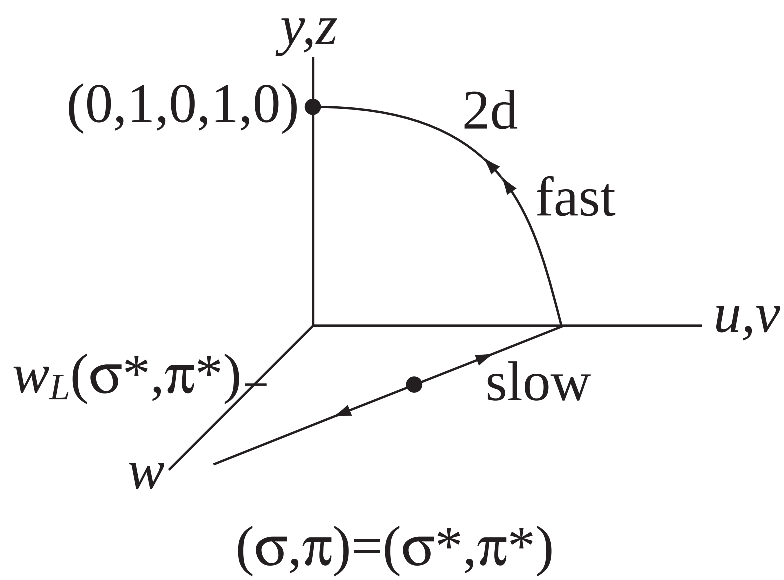
$$\sigma' = 0, \tag{0.39}$$

$$\pi' = 0. \tag{0.40}$$

For small $q > 0$:

- There is a 2-dimensional sheet of equilibria E_q in L_q given exactly by $w = w_L(\sigma, \pi)$.
- All equilibria in E_q have $u = v$ and are repelling within L_q.

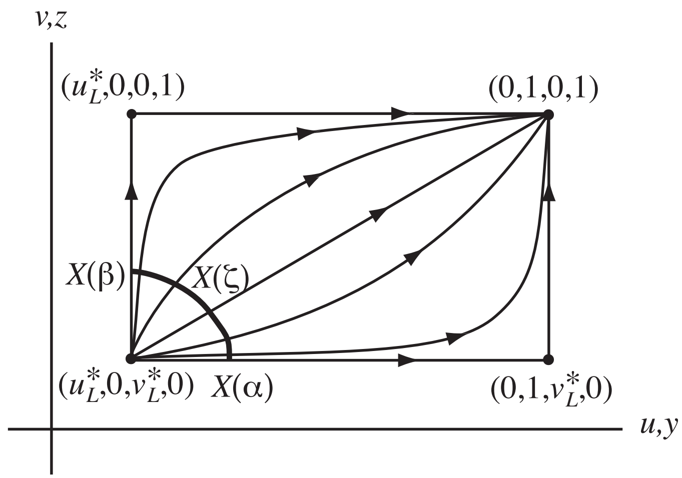
Solutions in $W^u(L_q) \cap W^s(R_q)$ are near a "singular solution":



Proceeding backward from $(0, 1, 0, 1, 0)$:

- Fast part: one of the connections in the last picture to $(u_L^*, 0, v_L^*, 0, 0)$ (i.e., a pair of one-layer combustion waves, perhaps shifted).
- Slow part: y and z remain 0 (the coke is burned), the temperatures slowly equilibrate (at a value between u_L^* and v_L^*).

To study small $q > 0$, parameterize the connecting orbits from $(u_L^*, 0, v_L^*, 0, 0)$ to $(0, 1, 0, 1, 0)$ for $(\sigma, \pi, q) = (\sigma^*, \pi^*, 0)$ by ζ, $\alpha \leq \zeta \leq \beta$:



Separation function between $W^u(L_q)$ and $W^s(R_q)$:

- S : $\zeta\sigma\pi q$-space $\to \mathbb{R}^2$.
- $S(\zeta, \sigma^*, \pi^*, 0) \equiv 0$

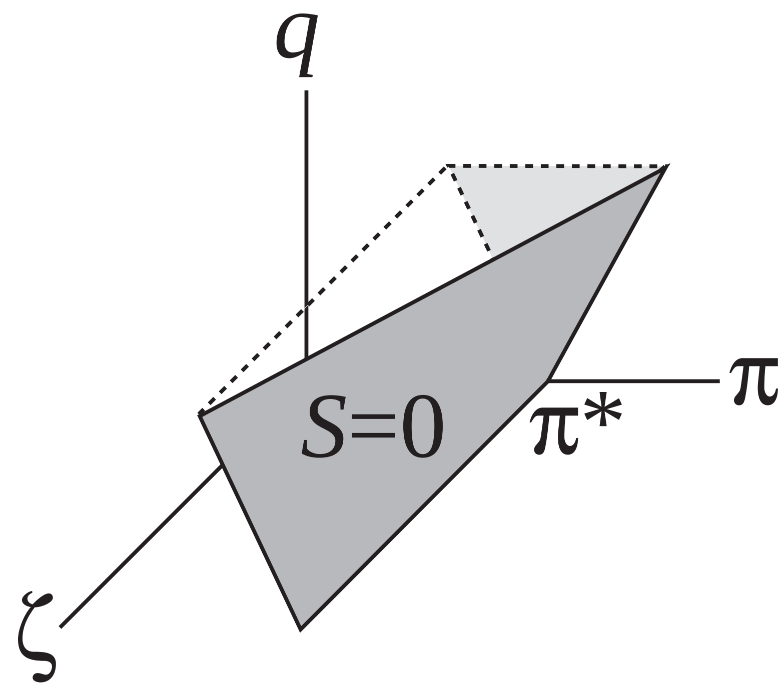
The implicit function lets us solve the equation

$$S(\zeta, \sigma, \pi, q) = 0$$

for (σ, π) in terms of (ζ, q) with

$$(\sigma(\zeta, 0), \pi(\zeta, 0)) = (\sigma^*, \pi^*).$$

Graph of $\pi(\zeta, q)$:



We wish to show that $\frac{\partial \pi}{\partial q}(\zeta, 0)$ is not constant.

Theorem 0.5. *As $\zeta \to \alpha$, $\frac{\partial \pi}{\partial q}(\zeta, 0) \to \infty$, and as $\zeta \to \beta$, $\frac{\partial \pi}{\partial q}(\zeta, 0) \to -\infty$; or the reverse. In particular, for small $q > 0$, the range of $\pi(\zeta, q)$ includes an open interval.*

Proof.

$$\frac{\partial \pi}{\partial q}(\zeta, 0) = \frac{M_q(\zeta)N_\sigma - M_\sigma N_q(\zeta)}{M_\sigma N_\pi - M_\pi N\sigma},$$

with each term a Melnikov integral. In particular,

$$M_q(\zeta) = \int_{-\infty}^{\infty} \psi_{15}^*(\xi)\frac{\partial F}{\partial q}\,d\xi = \int_{-\infty}^{\infty} \psi_{15}^*(\xi)\Big(v^*\big(\xi + \eta^*(\zeta)\big) - u^*(\xi)\Big)\,d\xi$$

and

$$N_q(\zeta) = \int_{-\infty}^{\infty} \phi_{25}^*(\eta)\frac{\partial F}{\partial q}\,d\eta = \int_{-\infty}^{\infty} \phi_{25}^*(\eta)\Big(v^*(\eta) - u^*\big(\eta - \eta^*(\zeta)\big)\Big)\,d\eta.$$

Near $\zeta = \alpha$, the uy-component of the orbit arrives near $(0, 1)$ while the vz-component is still near $(v_L^*, 0)$. The orbit spends a long time with (u, y, v, z) near $(0, 1, v_L^*, 0)$, where $v - u$ is large positive. Meanwhile ξ is very large, so $\psi_{15}^*(\zeta)$ is near a positive constant. The integral becomes large. □

The traveling wave associated with a nearby connecting orbit has the combustion zones in the two layers displaced.

Concluding remarks

Numerical results have been pretty well explained.

Stability of these waves has not yet been analyzed.

It may be possible to prove existence of a strong connection for any values of the system parameters using the Conley index.

Can other models be analyzed similarly?