MA 493B

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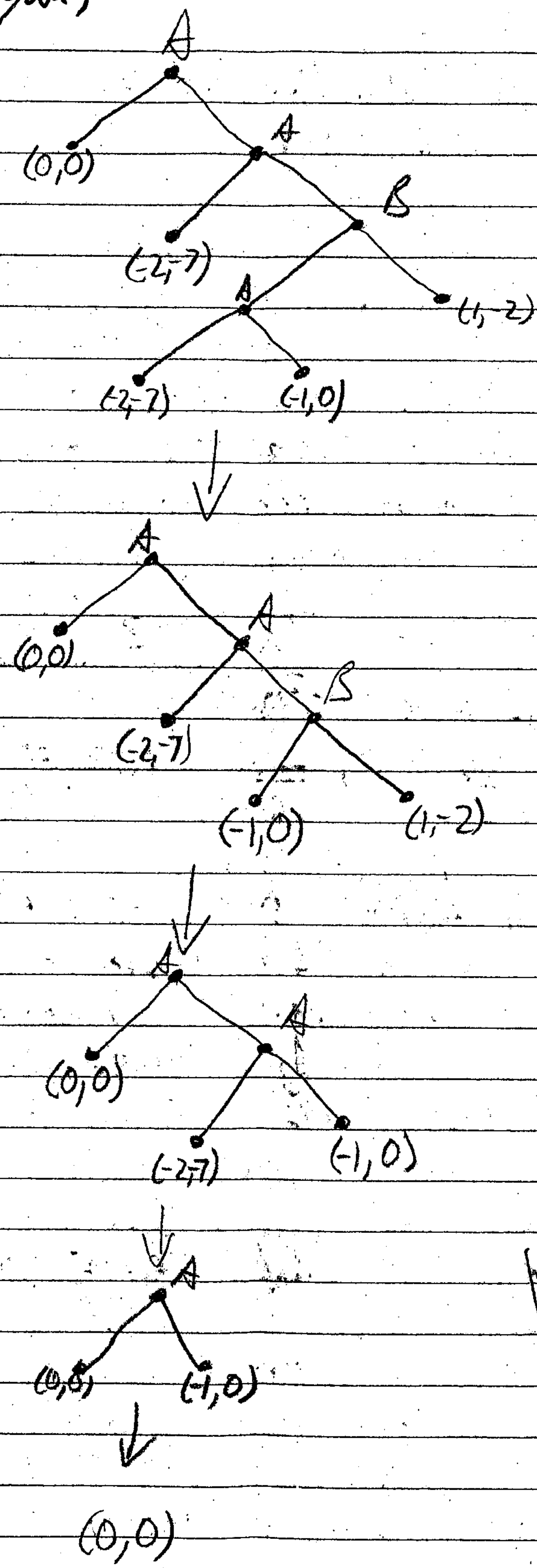
Test 1 Answers

(Thanks to Matthew Bryan)

1.



By backward induction, Alpha Technology should not start legal action.

2. $\frac{\partial r_2}{\partial x_2} = 10 - 2x_1 - 4x_2 = 0 \Rightarrow x_2 = \frac{10-2x_1}{4} = \frac{5-x_1}{2}$

Thus r_2 has a local maximum or minimum at $x_2 = \frac{5-x_1}{2}$. Since r_2 is a negative quadratic for fixed x_1, this must be a maximum. Thus $b(x_1) = \frac{5-x_1}{2}$

$r_1(x_1, b(x_1)) = 10x_1 - 2x_1^2 - 2x_1\left(\frac{5-x_1}{2}\right) = 10x_1 - 2x_1^2 - 5x_1 + x_1^2$
$= 5x_1 - x_1^2$

$\frac{d r_1(x_1, b(x_1))}{d x_1} = 5 - 2x_1 = 0 \Rightarrow x_1 = \frac{5}{2}$

Thus $r_1(x_1, b(x_1))$ has a local extreme at $x_1 = \frac{5}{2}$. Since $r_1(x_1, b(x_1))$ is a negative quadratic, this must be a maximum. Therefore, the best choose of x_1 for Ajax industries is $\frac{5}{2}$ by backward induction.

3. a) The payoff (2,6) corresponds to Al choosing the strategy Tit for Tat and Bob choosing the strategy of always having Low prices. In this case, Al will spend the first week with high prices and Bob will have low prices. This creates a payoff of (0,4) for the first week. In the second week, Al will switch to having low prices and Bob will still have low prices. This provides a payoff of (2,2) for

that week. Thus the total payoff for that week is $(0,4)+(2,2)=(2,6)$.

b) Al does not have any strictly dominated strategies

c) Al's strategy H is weakly dominated by both strategies L and T, and Al's strategy T is weakly dominated by L.

d) Bob

Al	H	L	T
~~H~~	~~(6,6)~~	~~(0,8)~~	~~(6,6)~~
L	(8,0)	(4,4)	(6,2)
~~T~~	~~(6,6)~~	~~(2,6)~~	~~(6,6)~~

First eliminate Al's H strategy, then Al's T strategy, then Bob's H strategy, and finally Bob's T strategy.

Therefore the strategy profile (L,L) is a Nash equilibrium.

e) Bob

Al	H	L	T
H	(6,6)	$(0,\boxed{8})$	$(\boxed{6},6)$
L	$(\boxed{8},0)$	$(\boxed{4},\boxed{4})$	$(\boxed{6},2)$
T	$(6,\boxed{6})$	$(2,\boxed{6})$	$(\boxed{6},\boxed{6})$

By best response, there are two Nash Equilibrium. One is the N.e. found by backward induction, (L,L). The second is the strategy set (T,T)

4.a) Yes, When $(s,t)=(0,1)$, the first child cannot improve his payoff. This is because if he selects $0<s<t$, then his payoff is $-s<0$ which is his payoff when $s=0$. If he chooses $s=1$, then $s=t$ and his payoff is $\frac{1}{2}-s=\frac{1}{2}-1=-\frac{1}{2}<0$. On the other hand if child two changes his strategy to $0<t<1$, then his payoff is still $1-s=1$ which is the same payoff for $t=1$. If the second child chooses $t=0$, then $s=t$ and his payoff is $\frac{1}{2}-s=\frac{1}{2}-0=\frac{1}{2}<1$. Therefore the strategy profile $(0,1)$ is a Nash equilibrium. Since the payoffs for each child is symmetric, the strategy set $(1,0)$ is a Nash equilibrium by a similar argument as before.

b) No, there is never a Nash equilibrium in this case since the first child can always improve his payoff by choosing $s>t$. This is because his payoff for $s<t$ is $-s<0<1-t$ which is his payoff for $t<s$.

c) No. When $s=t$, the second child can improve his payoff from $\frac{1}{2}-s$ to $1-s$ by choosing a $t>s$. Since $\frac{1}{2}-s<1-s$ $\forall\, 0\leq s\leq 1$. This includes the strategy profile $(0,0)$ ~~since~~ child two can change to $t=1$ and get a payoff of 1. For the strategy profile $(1,1)$, child two can change to a value of $t=0$ and improve his payoff from $\frac{1}{2}-1=-\frac{1}{2}$ to 0. Thus $s=t$ has no Nash equilibria

Points ① 22 ② 22 ③ 28 ④ 28