MA 493B Test 1

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1. Alpha Technology claims that Beta Technology used Alpha's ideas in one of its products. Alpha is considering suing Beta. Alpha is facing legal costs of \$100K to start legal action, plus an additional \$400K in legal costs if the suit goes to trial. Beta also faces \$400K in legal costs if the suit goes to trial. If Alpha wins, it expects to collect \$3M from Beta. However, the probability that Alpha will win if the suit goes to trial is only 10%, so Alpha's expected winnings if the suit goes to trial are only \$300K. Alpha's lawyer suggests starting legal action, then making a settlement offer: Alpha will offer to drop the lawsuit in exchange for a payment of \$200K from Beta.

   The following tree diagram shows the situation.

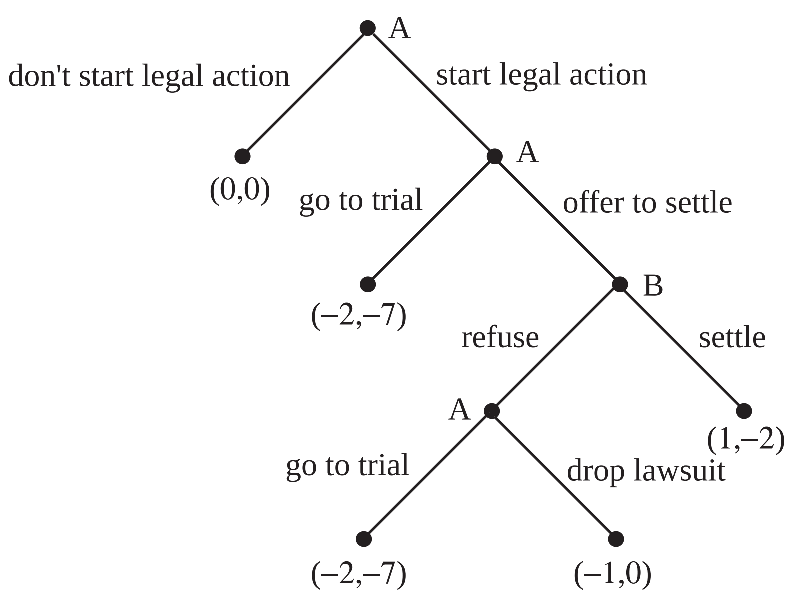

Figure 1: A is Alpha, B is Beta. Payoffs are shown in units of \$100K.

Use backward induction to figure out what Alpha should do. Be sure I can follow your reasoning.

2. Ajax Industries and Babar Industries both produce widgets. Let x_1 be the quantity produced by Ajax and let x_2 be the quantity produced by Babar. The price of widgets is

$$p = 10 - 2(x_1 + x_2).$$

Thus the more widgets the two companies produce, the lower will be the price. (This formula can produce a negative price; don't worry about it.)

The revenue of each company is the price times the quantity if produces. Thus the revenue of Ajax Industries is

$$r_1 = px_1 = 10x_1 - 2x_1^2 - 2x_1x_2,$$

and the revenue of Babar Industries is

$$r_2 = px_2 = 10x_2 - 2x_1x_2 - 2x_2^2.$$

We regard this as a two-person game. The players choose x_1 and x_2; the payoffs are r_1 and r_2.

Suppose Ajax chooses x_1 first, then Babar observes x_1 and chooses x_2. Use backward induction to find Ajax's best choice.

3. There are two toy stores in town, Al's and Bob's. If both charge high prices, both make \$3K per week. If both charge low prices, both make \$2K per week. If one charges high prices and one charges low prices, the one that charges high prices makes nothing, and the one that charges low prices makes \$4K per week.

At the start of each week, both stores independently set their prices for the week.

We will consider three possible strategies for each store:

- H: Always charge high prices.
- L: Always charge low prices.
- T: Tit-for-tat. Charge high prices the first week. The next week, do whatever the other store did the previous week.

The following table shows the payoffs if each store follows its strategy for two weeks.

		Bob		
		H	**L**	**T**
Al	**H**	(6,6)	(0,8)	(6,6)
	L	(8,0)	(4,4)	(6,2)
	T	(6,6)	(2,6)	(6,6)

(a) Explain the $(2, 6)$ payoffs in the third line of the table.

(b) Which of Al's strategies are strictly dominated?

(c) Which of Al's strategies are weakly dominated?

(d) Use iterated elimination of weakly dominated strategies to find a Nash equilibrium.

(e) Use best response to find all Nash equilibria.

4. Two children begin to argue about some marbles with a value of 1. If one child gives up arguing first, the other child gets the marbles. If both children give up arguing at the same time, they split the marbles.

The payoff to each child is the value of the marbles he gets, minus the length of time in hours that the argument lasts.

After one hour, it will be time for dinner. If the argument has not ended before then, it ends then, and the children split the marbles.

We consider this a two-person game. Before the game begins, each child decides independently how long he is willing to argue, in hours. Thus the first child's strategy is a number s, $0 \leq s \leq 1$, and the second child's strategy is a number t, $0 \leq t \leq 1$.

The payoffs are:

- If $s < t$, the argument ends after s hours and the second child gets the marbles, so $\Pi_1(s, t) = -s$ and $\Pi_2(s, t) = 1 - s$.
- If $s > t$, the argument ends after t hours and the first child gets the marbles, so $\Pi_1(s, t) = 1 - t$ and $\Pi_2(s, t) = -t$.
- If $s = t$, the argument ends after s hours and the children split the marbles, so $\Pi_1(s, t) = \frac{1}{2} - s$ and $\Pi_2(s, t) = \frac{1}{2} - s$.

(a) Are $(s, t) = (0, 1)$ and $(s, t) = (1, 0)$ Nash equilibria? Explain.

(b) Are there any Nash equilibria (s, t) with $0 < s < t < 1$? Explain.

(c) Are there any Nash equilibria with $s = t$? Explain. Make sure you have dealt with $(0, 0)$ and $(1, 1)$.