MA 440 Practice Final Answers

① a) Need $\frac{\partial \pi_1}{\partial x_1} = 0$ and $\frac{\partial \pi_2}{\partial x_2} = 0$

$$\left.\begin{aligned} \frac{\partial \pi_1}{\partial x_1} &= 97 - 4x_1 - 2x_2 = 0 \\ \frac{\partial \pi_2}{\partial x_2} &= 98 - 2x_1 - 4x_2 = 0 \end{aligned}\right\}$$

Solve simultaneously: $(x_1, x_2) = (16, 16\tfrac{1}{2})$

b) Baban's best response to x_1:

$$\frac{\partial \pi_2}{\partial x_2} = 98 - 2x_1 - 4x_2 = 0 \Rightarrow x_2 = \frac{98-2x_1}{4} = \frac{49-x_1}{2}$$

Ajax's payoff if Baban uses best response:

$$\pi_1\left(x_1, \frac{49-x_1}{2}\right) = 97x_1 - 2x_1^2 - 2x_1\left(\frac{49-x_1}{2}\right) = 48x_1 - x_1^2$$

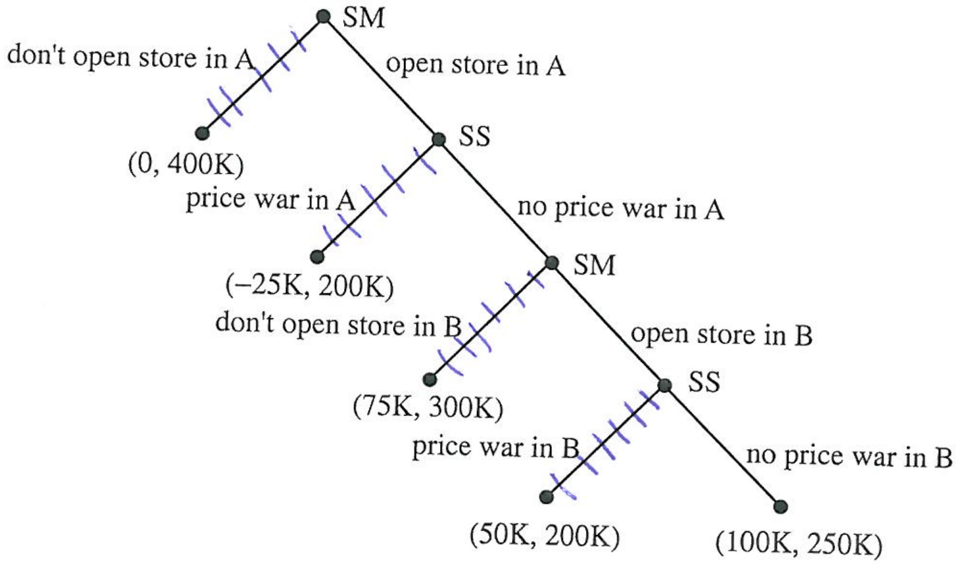
This is maximum when $\frac{d}{dx_1}\pi_1\left(x_1, \frac{49-x_1}{2}\right) = 48 - 2x_1 = 0$

$\Rightarrow x_1 = 24$

② a)

	A YR 1	A YR 2	B YR 1	B YR 2
SM	25	50	0	-25
SS	50	50	100	0

Total is 50 for SM, 200 for SS



SM should open store in A, SS no price war in A,
SM " " " " B, SS " " " " B.

(3) a) There are no Nash equilibria with $s<t$. Player 2 can improve his payoff be reducing t a little, but still keeping it above s.

b) No strategy profile (t,t) with $t<1$ is a Nash equilibrium. This choice of strategies gives player 2 a payoff of $1-t$. If player 2 changes to t' with $t<t'<1$, his new payoff will be $2-(t+t')$. This is greater than $1-t$:

$$2-(t+t') > 1-t$$
$$\iff \quad 2-t' > 1$$
$$\iff \quad 1 > t'$$

On the other hand (1,1) <u>is</u> a Nash eq. It gives each player a payoff of 0. If one player changes to a lower number, his payoff is still 0.

4 a) (20,80) : If C1 produces level 1 and C2 produces level 2, C1 will sell to those who prefer level 1 (20%), and C2 to those who prefer levels 2, 3, 4, 5 (80%).

(50,50) : If both produce level 2, everybody chooses randomly which cereal to buy, so each company gets 50% of the market.

etc.

		3	7	Company 2	8	4	
		1	2	3	4	5	
	~~1~~	~~(50,50)~~	~~(20,80)~~	~~(30,70)~~	~~(40,60)~~	~~(50,50)~~	1
	2	(80,20)	~~(50,50)~~	~~(40,60)~~	~~(50,50)~~	(60,40)	5
Company1	3	(70,30)	(60,40)	(50,50)	(60,40)	(70,30)	
	4	(60,40)	~~(50,50)~~	~~(40,60)~~	~~(50,50)~~	(80,20)	6
	~~5~~	~~(50,50)~~	~~(40,60)~~	~~(30,70)~~	~~(20,80)~~	~~(50,50)~~	2

. . .

1 Eliminate C1 strategy 1, strictly dominated by strat. 2 or 3
2 " " " 5, " " " " 4 or 3
3 " C2 " 1, " " " " 2 or 3
4 " " " 5, " " " " 4 or 3
5 " C1 " 2, " " " " 3
6 " " " 4, " " " " 3
7 " C2 " 2, " " " " 3
8 " " " 4, " " " " 3

Nash equilibrium: $(3,3)$.

(5) a)

		Driver 2		
		T	W	C
Driver 1	T	$(-5,-5)$	$(\boxed{0},\boxed{-2})$	$(\boxed{0},\boxed{-2})$
	W	$(\boxed{-2},\boxed{0})$	$(-3,-3)$	$(-2,\boxed{0})$
	C	$(\boxed{-2},\boxed{0})$	$(\boxed{0},-2)$	$(-5,-5)$

(T,W), (T,C), (W,T), (C,T)

b) Let $\sigma = p_1 T + p_2 W + p_3 C$, $\tau = q_1 T + q_2 W + q_3 C$

		q_1	q_2	q_3
		T	W	C
p_1	T	$(-5,-5)$	$(0,-2)$	$(0,-2)$
p_2	W	$(-2,0)$	$(-3,-3)$	$(-2,0)$
p_3	C	$(-2,0)$	$(0,-2)$	$(-5,-5)$

If (σ,τ) is a Nash equilibrium and all $p_i > 0$, the following are equal:

$$\Pi_1(T,\tau) = -5q_1 + 0q_2 + 0q_3 = -5q_1$$
$$\Pi_1(W,\tau) = -2q_1 - 3q_2 - 2q_3$$
$$\Pi_1(C,\tau) = -2q_1 + 0q_2 - 5q_3 = -2q_1 - 5q_3$$

Using the equations $\Pi_1(T,\tau) = \Pi_1(W,\tau)$ and $\Pi_1(T,\tau) = \Pi_1(C,\tau)$ we have

$$3q_1 - 3q_2 - 2q_3 = 0$$
$$3q_1 - 5q_3 = 0$$

We add the equation

$$q_1 + q_2 + q_3 = 1$$

and solve the resulting system of 3 equations in 3 unknowns:

$$(q_1, q_2, q_3) = \left(\frac{5}{11}, \frac{3}{11}, \frac{3}{11}\right)$$

Since we'll get the same equations for (p_1, p_2, p_3) if we assume all $q_i > 0$, we also have

$$(p_1, p_2, p_3) = \left(\frac{5}{11}, \frac{3}{11}, \frac{3}{11}\right).$$

c) Let $\sigma = p_1 T + p_3 C$, $\tau = q_1 T + q_3 C$

If (σ, τ) is a Nash equilibrium and $p_1 > 0$, $p_3 > 0$, then

$$\Pi_1(T, \tau) = \Pi_1(C, \tau)$$

$$-5q_1 + 0q_3 = -2q_1 - 5q_3$$

$$\Rightarrow \quad -3q_1 + 5q_3 = 0$$

Second equation: $q_1 + q_3 = 1$

Solve: $(q_1, q_3) = \left(\frac{5}{8}, \frac{3}{8}\right)$

Since we'll get the same equations for (p_1, p_3) if we assume $q_1 > 0$, $q_3 > 0$, we also have $(p_1, p_3) = \left(\frac{5}{8}, \frac{3}{8}\right)$.

Is (σ, τ) a Nash equilibrium? We also need

$$\Pi_1(W, \tau) \leq \Pi_1(T, \tau) \text{ (which equals } \Pi_1(C, \tau))$$

$$\Pi_1(W, \tau) = -2q_1 - 2q_3 = -2 \cdot \tfrac{5}{8} - 2 \cdot \tfrac{3}{8} = -2$$
$$\Pi_1(T, \tau) = -5q_1 + 0q_3 = -5 \cdot \tfrac{5}{8} = -\tfrac{25}{8} = -3\tfrac{1}{8}$$

Thus $\Pi_1(W, \tau) > \Pi_1(T, \tau)$, so (σ, τ) is <u>not</u> a Nash equilibrium.

(6) Assume SM uses the trigger strategy. If SS first charges low in period k, it gets 6 in period k. SM will then charge low in every subsequent period. SS's best response is to also charge low. Thus the most SS can hope to make is

$$6\delta^k + 3\delta^{k+1} + 3\delta^{k+2} + \dots = 6\delta^k + \frac{3\delta^{k+1}}{1-\delta}$$

If SS had never charged low, its payoff from period k on would have been

$$4\delta^k + 4\delta^{k+1} + \dots = \frac{4\delta^k}{1-\delta}.$$

Therefore it is a Nash eq. for both to use the trigger strategy provided

$$\frac{4\delta^k}{1-\delta} \geq 6\delta^k + \frac{3\delta^{k+1}}{1-\delta}$$

Multiply by $\frac{1-\delta}{\delta^k}$:
$$4 \geq 6(1-\delta) + 3\delta$$
$$4 \geq 6 - 3\delta$$
$$3\delta \geq 2 \rightarrow \underline{\underline{\delta \geq \tfrac{2}{3}}}$$

7 a) If expert diagnoses major repair as major, his payoff is $M+B$ or B, hence at least B. If expert diagnoses major repair as minor, his payoff is m or 0, hence at most m. Since $B > m$, he should diagnose a major repair as major.

b) Payoffs when problem is major (so expert always says its major):

Ex	Rr	Rn	Nr	Nn
D	$(M+B, V-M)$	$(M+B, V-M)$	$(B,0)$	$(B,0)$
H	$(M+B, V-M)$	$(M+B, V-M)$	$(B,0)$	$(B,0)$

Payoffs when problem is minor:

Ex	Rr	Rn	Nr	Nn
D	$(M, v-M)$	$(M, v-M)$	$(0,0)$	$(0,0)$
H	$(m+B, v-m)$	$(B, 0)$	$(m+B, v-m)$	$(B,0)$

Total payoffs are average of these matrices:

	Rr	Rn	Nr	Nn
D	$\left(M+\frac{1}{2}B, \frac{v+V}{2}-M\right)$	$\left(M+\frac{1}{2}B, \frac{v+V}{2}-M\right)$	$\left(\frac{1}{2}B, 0\right)$	$\left(\frac{1}{2}B, 0\right)$
H	$\left(\frac{m+M}{2}+B, \frac{v+V-m-M}{2}\right.$	$\left(\frac{1}{2}M+B, \frac{V-M}{2}\right.$	$\left(\frac{1}{2}m+B, \frac{v-m}{2}\right)$	$(B,0)$

(8) a)

		q climb	$1-q$ wait
p	climb	$(2,2)$	$(\boxed{1},\boxed{4})$
$1-p$	wait	$(\boxed{4},\boxed{1})$	$(0,0)$

(w,c) and (c,w) are Nash eq. They are not symmetric

b) $\sigma = pC + (1-p)W$, $\tau = qC + (1-q)W$

If $0<p<1$: $\pi_1(C,\tau) = \pi_1(W,\tau)$

$$2q + 1(1-q) = 4q + 0(1-q)$$

$$1 = 3q$$

$$q = \frac{1}{3}$$

Similarly, if $0<q<1$, then $p = \frac{1}{3}$.

$(\frac{1}{3}C + \frac{2}{3}W, \frac{1}{3}C + \frac{2}{3}W)$ is a mixed-strategy Nash eq.

c) When each strategy is the best response to the other (as here), the pop state corresponding to the mixed strategy Nash eq. is e.v. stable.

d) Let $\sigma = p_1 C + p_2 W$

$$\pi_{1\sigma} = 2p_1 + 1p_2$$
$$\pi_{2\sigma} = 4p_1$$
$$\pi_{\sigma\sigma} = p_1\pi_{1\sigma} + p_2\pi_{2\sigma} = 2p_1^2 + 5p_1p_2$$

$$\dot{p}_1 = (\pi_{1\sigma} - \pi_{\sigma\sigma})p_1 = (2p_1 + p_2 - 2p_1^2 - 5p_1p_2)p_1$$

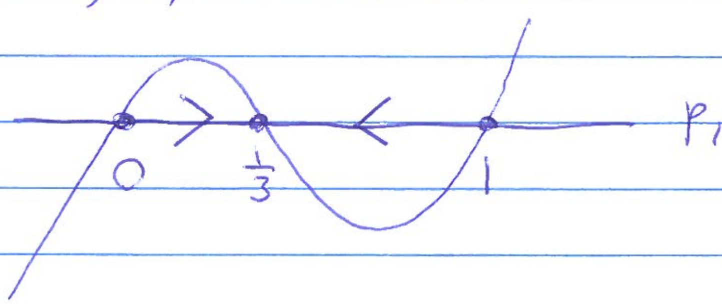
$$\dot{p}_2 = (\pi_{2\sigma} - \pi_{\sigma\sigma})p_2 = (4p_1 - 2p_1^2 - 5p_1p_2)p_2$$

Reduced to one equation: let $p_2 = 1 - p_1$ in $\dot{p}_1$ equation:

$$\dot{p}_1 = (2p_1 + 1 - p_1 - 2p_1^2 - 5p_1(1-p_1))\,p_1$$
$$= (1 - 4p_1 + 3p_1^2)p_1 = (1-p_1)(1-3p_1)\,p_1$$

e) $p_1 = 1, \frac{1}{3}, 0$

f)



(9) a)

	climb (p)	wait (q)	shake (r)
climb	$(0,0)$	$(\boxed{1},\boxed{3})$	$(0,0)$
wait	$(\boxed{3},\boxed{1})$	$(0,0)$	$(0,0)$
shake	$(0,0)$	$(0,0)$	$(\boxed{2},\boxed{2})$

(C,W), (W,C), (S,S) are Nash eq. (S,S) is symmetric.

b) $\Pi_{1\sigma} = 1 \cdot q = q$

$\Pi_{2\sigma} = 3p$

$\Pi_{3\sigma} = 2r$

$\Pi_{\sigma\sigma} = p\Pi_{1\sigma} + q\Pi_{2\sigma} + r\Pi_{3\sigma} = 4pq + 2r^2$

$$\dot{p} = (\Pi_{1\sigma} - \Pi_{\sigma\sigma})p = (q - 4pq - 2r^2)p$$
$$\dot{q} = (\Pi_{2\sigma} - \Pi_{\sigma\sigma})q = (3p - 4pq - 2r^2)q$$

not needed $\rightarrow$ $\dot{r} = (\Pi_{3\sigma} - \Pi_{\sigma\sigma})r = (2r - 4pq - 2r^2)r$

Use just $\dot{p}$ and $\dot{q}$ equations with $r = 1-p-q$:

$$\dot{p} = (q - 4pq - 2(1-p-q)^2)\, p$$
$$\dot{q} = (3p - 4pq - 2(1-p-q)^2)\, q$$

c) We must solve the pair of equations $\dot{p}=0$ and $\dot{q}=0$ with $p+q=1$. Substituting $q = 1-p$ into these equations gives

$$(1-p-4p(1-p) - 2\cdot 0^2)\,p = 0$$
$$(3p - 4p(1-p) - 2\cdot 0^2)(1-p) = 0$$

$$(1-5p+4p^2)\,p = 0$$
$$(-p+4p^2)(1-p) = 0$$

$$(1-p)(1-4p)\,p = 0$$
$$p(-1+4p)(1-p) = 0$$

We can make both equations true by taking $p = 0, \frac{1}{4}$, or 1.
So the points are $(p,q) = (0,1), \left(\frac{1}{4}, \frac{3}{4}\right), (1,0)$

d) $$\begin{bmatrix} \frac{\partial \dot p}{\partial p} & \frac{\partial \dot p}{\partial q} \\ \frac{\partial \dot q}{\partial p} & \frac{\partial \dot q}{\partial q} \end{bmatrix} = \begin{bmatrix} -\frac{\partial h}{\partial p} \cdot p + q - h & \left(1 - \frac{\partial h}{\partial q}\right) p \\ \left(3 - \frac{\partial h}{\partial p}\right) q & -\frac{\partial h}{\partial q} \cdot q + 3p - h \end{bmatrix}$$

Set $p = q = 0$:

$$\begin{bmatrix} -h(0,0) & 0 \\ 0 & -h(0,0) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

Eigenvalues are both -2 : attractor.

e) If you start above the stable manifold of the interior equilibrium, you converge to $(p,q) = \left(\frac{1}{4}, \frac{3}{4}\right)$ found in part c. If you start below the stable manifold of the interior equilibrium, you converge to $(p,q) = (0,0)$.

At $(p,q) = \left(\frac{1}{4}, \frac{3}{4}\right)$, $\frac{1}{4}$ climb, $\frac{3}{4}$ wait, no one shakes.
At $(p,q) = (0,0)$, no one climbs, no one waits, everyone shakes.