MA 225   Test 3 Review Questions

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1. Define a relation R on $\mathbb{R}$ by $x\,R\,y$ iff $\sin x = \sin y$. Show that R is reflexive, symmetric and transitive.

2. Let X denote the set of all open intervals. (A typical element of X is an interval of the form (a, b).) Define a relation R on X by $A\,R\,B$ iff A and B are disjoint intervals. Explain why R is not reflexive or transitive, but is symmetric.

3. Define an equivalence relation on $\mathbb{N} \times \mathbb{N}$ by $(a, b)\,R\,(c, d)$ iff $a + b = c + d$. List all elements in the equivalence class of $(2, 4)$.

4. Define $f : \mathbb{Z}_2 \to \mathbb{Z}_4$ by $f(x/\equiv_2) = x/\equiv_4$. Is this a function? Explain.

5. Define $f : \mathbb{R} - \{2\} \to \mathbb{R}$ by $f(x) = \frac{x-1}{x-2}$. Find the range of f.

6. Define $f : \mathbb{R} \to \mathbb{R}$ by
$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \,, \\ 4x & \text{if } x < 0. \end{cases}$$
   Show that f is one-to-one and onto, and find the inverse function.

7. Define the following functions:

$$f : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N} \qquad\qquad f(x, y) = (x^2, y)$$
$$g : \mathbb{N} \to \mathbb{N} \qquad\qquad g(x) = x^2$$
$$\Pi_1 : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \qquad\qquad \Pi_1(x, y) = x$$

   (a) Find the domain and codomain of $\Pi_1 \circ f$.
   (b) Find the domain and codomain of $g \circ \Pi_1$.
   (c) Show that $\Pi_1 \circ f = g \circ \Pi_1$.

8. Let A, B and C be sets, let $f : A \to C$ be a function, and let $\Pi_1 : A \times B \to A$ be projection.

   (a) Show that if f is onto, then $f \circ \Pi_1$ is onto.
   (b) Suppose f is one-to-one. Under what conditions on B is $f \circ \Pi_1$ one-to-one?