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Test 2 Answers

① Assume $A \subseteq B$ and $C \subseteq \tilde{D}$

Let $x \in A \cap D$. Then $x \in A$ and $x \in D$.

Since $x \in A$ and $A \subseteq B$, $x \in B$.

Since $x \in D$ and $C \subseteq \tilde{D}$, $x \notin C$.

(Reason: Since $C \subseteq \tilde{D}$, $x \in C \Rightarrow x \in \tilde{D}$, so $x \in C \Rightarrow x \notin D$, so $x \in D \Rightarrow x \notin C$.)

Thus $x \in B$ and $x \notin C$, so $x \in B - C$.

Thus $A \cap D \subseteq B - C$.

② Assume $B \subseteq A$. Suppose $\exists x$ such that $x \in \tilde{A} \cap B$. Then $x \in \tilde{A}$ and $x \in B$.

Since $x \in \tilde{A}$, $\underline{x \notin A}$. Since $x \in B$ and $B \subseteq A$, $\underline{x \in A}$.

This is a contradiction. Therefore $\tilde{A} \cap B = \emptyset$.

③ Assume that for every $A \in \mathcal{Q}$, $A \subseteq B$ or $A \subseteq C$.

Let $x \in \bigcup_{A \in \mathcal{Q}} A$. Then $\exists A \in \mathcal{Q}$ such that $x \in A$.

Case 1. $A \subseteq B$. Then $x \in B$, so $x \in B \cup C$.

Case 2. $A \subseteq C$. Then $x \in C$, so $x \in B \cup C$.

In both cases, $x \in B \cup C$. Therefore $\bigcup_{A \in \mathcal{Q}} A \subseteq B \cup C$.

(4) $\bigcap_{n=1}^{\infty} A_n = [0,1)$ $\bigcup_{n=1}^{\infty} A_n = (-1,\infty)$

(5) (i) $n=1$: $3\cdot 1 - 1 = \frac{1}{2}\cdot 1\,(3\cdot 1+1)$?

$2 = \frac{1}{2}\cdot 4$ ✓

(ii) Assume that for some n, $2+5+8+\cdots+(3n-1) = \frac{1}{2}n(3n+1)$

We must show that

$$2+5+8+\cdots+\big(3(n+1)-1\big) = \frac{1}{2}(n+1)\big(3(n+1)+1\big)$$

$$= \frac{1}{2}(n+1)(3n+4)$$

$$2+5+8+\cdots+3(n+1)-1 = \frac{1}{2}n(3n+1) + (3n+2)$$

$$= \frac{1}{2}\left(3n^2+n+6n+4\right) = \frac{1}{2}\left(3n^2+7n+4\right) = \frac{1}{2}(n+1)(3n+4)$$

(iii) By PMI, for every $n\in\mathbb{N}$, $2+5+8+\cdots+(3n-1) = \frac{1}{2}n(3n+1)$

(6) (i) $n=6$: $2^6 > (1+6)^2$? $64>49$ ✓

(ii) Assume that for some $n\geq 6$, $2^n > (1+n)^2$. We must show that $2^{n+1} > (1+(n+1))^2 = (2+n)^2$.

$$2^{n+1} = 2\cdot 2^n > 2\cdot(1+n)^2 = 2(1+2n+n^2) = 2+4n+2n^2$$
$$= (2+n^2)+4n+n^2 > 4+4n+n^2 = (2+n)^2.$$

(iii) By PMI, for every $n\geq 6$, $2^n > (1+n)^2$.