MA 225-001 S. Schecter

Test 1 Answers

Thanks to June Kim and T.J. Brannon for the answers.

1. Use truth table

P	Q	$\sim Q$	$\sim Q \Rightarrow P$	$P \vee Q$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	F

$\therefore$ they are equivalent.

(2) Everyone is free or everyone is not free

(1) $\forall x$(x is free) $\vee$ $\forall x$(x is not free)

(2) $\sim[\forall x$(x is free) $\vee$ $\forall x$(x is not free)$]$

$\sim\forall x$(x is free) $\wedge$ $\sim\forall x$(x is not free)

$\exists x \sim$(x is free) $\wedge$ $\exists x \sim$(x is not free)

$\longrightarrow$ $\exists x$(x is not free) $\wedge$ $\exists x$(x is free)

(3) Some people are not free and Some people are free.

3.

(a) If a divides b and $a+b$ devides c, then a divides c. (Integers)

∴ Assume: a divides b and $a+b$ devides c
then ∴ $\exists k$ such that $ak=b$ and $\exists m$ such that $(a+b)m=c$
then $(a+ak)m=c$ (substitution)
$am+akm=c$
$a(m+km)=c$
since m is an integer and km is an integer, $m+km$ is also an integer
∴ a divides c.

(b) If 6 does not divide $12a+3b$, then b is odd
(Integers, contrapositive proof)
$P \Rightarrow Q$ $\sim Q \Rightarrow \sim P$
Assume: b is even
Then $\exists k$ such that $b=2k$
∴ Then $12a+3b = 12a+3(2k)$
$= 12a+6k$
$= 6(2a+k)$
since $2a+k$ is an integer,
6 divides $6(2a+k)$
and 6 divides $12a+3b$.
∴ If 6 does not divide $12a+3b$, then b is odd

(C) If x is a rational number other than 0 and y is irrational, then xy is irrational (Real #'s, Proof by contradiction) $\sim(P \Rightarrow Q)$ $\quad P \wedge \sim Q$

Assume: x is a rational number $\neq 0$ and y is irrational and xy is rational

Then $\exists$ integers a and b such that ($a \neq 0$ and $b \neq 0$)

$$x = \frac{a}{b}$$

also $\exists$ integers c and d such that

$$xy = \frac{c}{d} \quad d \neq 0$$

Substituting $\frac{a}{b}$ for x we get

$$\frac{a}{b} y = \frac{c}{d}$$

$$y = \frac{bc}{ad}$$

since $a \neq 0$ & $d \neq 0$, $\frac{bc}{ad}$ is a rational number.

$\therefore$ y is a rational number.

This contradicts our assumption.

$\therefore$ If x is a rational number other than 0 and y is irrational, then xy is irrational

(d) If n is an integer, then $2-3n+n^2$ is even.

(proof by case)

Assume: n is an integer.

Case 1

n is odd

then $\exists$ integer k such that $n = 2k+1$

then $2-3n+n^2 = 2-3(2k+1)+(2k+1)^2$

$= 2-6k-3+4k^2+4k+1$

$= 4k^2-2k$

$= 2(2k^2-k)$

since $2k^2-k$ is an integer, $2(2k^2-k)$ is even.

$\therefore$ $2-3n+n^2$ is even.

Case 2

n is even

then $\exists$ integer m such that $n=2m$

then $2-3n+n^2 = 2-6m+4m^2$

$= 2(2m^2-3m+1)$

since $2m^2-3m+1$ is an integer, $2(2m^2-3m+1)$ is even.

$\therefore$ $2-3n+n^2$ is even.

$\therefore$ The statement is true.

e) If there is an integer k such that $n = 3k+1$, then there is an integer l such that $n+1 = 3l-1$

Assume there is an integer k s.t. $n = 3k+1$

then $n+1 = 3k+1+1$

$= 3k+2$

$= 3(k+1)-1$

suppose $l = k+1$ (since k is integer l is also an integer) then $n+1 = 3(k+1)-1$

$= 3l-1$

$\therefore$ This statement is true