MA 225-001 Test 1

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February 16, 2005

1. Use truth tables to decide whether $\sim Q \Rightarrow P$ and $P \vee Q$ are equivalent propositional forms.

2. (1) Translate into a symbolic sentence with quantifiers, (2) write a clear denial as a symbolic sentence, and (3) translate your denial into ordinary English. The universe is people.

   Everyone is free or everyone is not free.

3. Definitions:

   (a) Let a and b be integers. We say a *divides* b if there is an integer k such that $ak = b$.

   (b) An integer a is *even* if there is an integer k such that $a = 2k$.

   (c) An integer a is *odd* if there is an integer k such that $a = 2k + 1$.

   (d) A real number x is *rational* if there exist integers a and b, with $b \neq 0$, such that $x = \frac{a}{b}$.

   (e) A real number x is *irational* if it is not rational.

   Using these definitions, prove *four* of the following five statements. Do not use previously proved results. For some statements, the universe is given in parentheses. A suggested method of proof is given for some statements.

   (a) If a divides b and $a + b$ divides c, then a divides c. (Integers.)

(b) If 6 does not divide $12a + 3b$, then b is odd. (Integers. Contrapositive proof.)

(c) If x is a rational number other than 0 and y is irrational, then xy is irrational. (Real numbers. Proof by contradiction.)

(d) If n is an integer, then $2 - 3n + n^2$ is even. (Proof by cases.)

(e) For every integer n the following is true: If there is an integer k such that $n = 3k + 1$, then there is an integer l such that $n + 1 = 3l - 1$.