

MA 225-001 Test 3

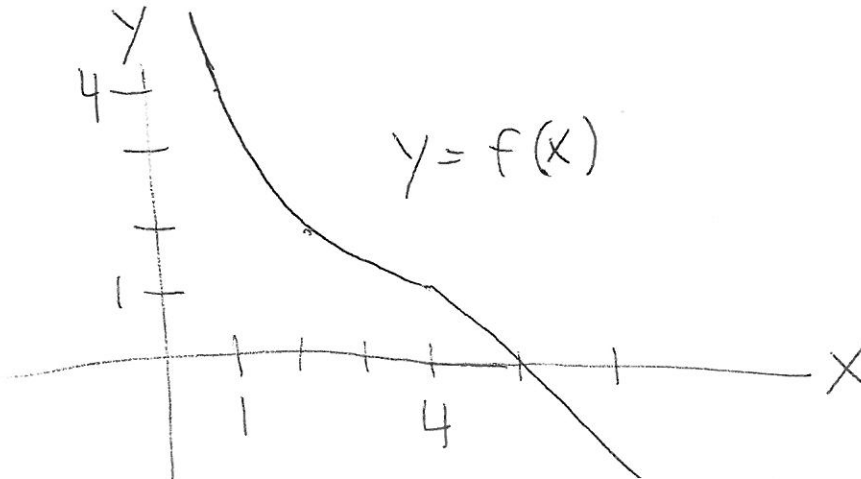
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1. Define a relation R on $\mathbb{R} \times \mathbb{R}$ by $(x, y) R (u, v)$ iff $xy = uv$.
 - (a) Prove that R is reflexive, symmetric and transitive.
 - (b) List two elements of the equivalence class of $(1, 4)$, other than $(1, 4)$ itself.
2. Define $f : \mathbb{Z}_2 \rightarrow \mathbb{Z}_3$ by $f(x/\equiv_2) = 2x/\equiv_3$. Is this a function? Justify your answer.
3. Define $f : \mathbb{Z}_8 \rightarrow \mathbb{Z}_4$ by $f(x/\equiv_8) = x/\equiv_4$.
 - (a) Prove that f is a function.
 - (b) Is f one-to-one? Briefly justify your answer.
4. Define $f : (0, \infty) \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{4}{x} & \text{if } 0 < x < 4. \\ 5 - x & \text{if } x \geq 4, \end{cases}$$

Show that f is one-to-one and onto.



5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $[0, \infty) \subseteq \text{Rng}(f)$. Let $g : \mathbb{R} \rightarrow [0, \infty)$ be the function $g(y) = y^2$. Prove that $g \circ f$ is onto.